Reconstructing Gravitational Wave Signals with Information Field Theory

Sebastián Gil Rodríguez
IMPRS Recruitment Workshop
8:30 AM February 6th, 2024

Outline

- Preamble

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- Preamble
- Laser Interferometers

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- Gravitational Wave Data

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- Implementation Details

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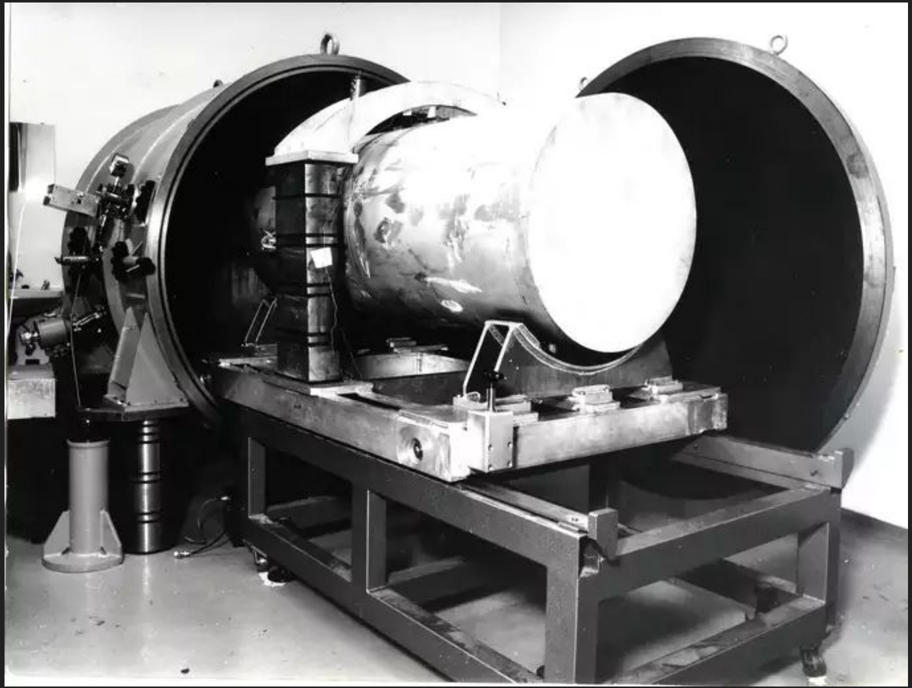
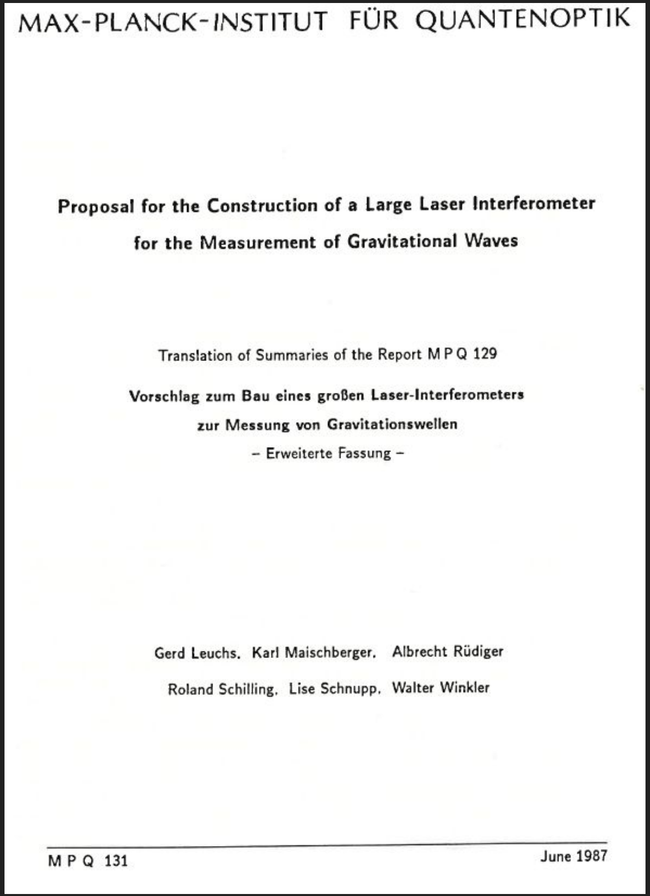
- Laser Interferometers
- Gravitational Wave Data
- Information Field Theory
- Implementation Details
- Results

Preamble



Left: the Munich resonant-mass gravitational detector (1972-1975)
Right: MPQ’s 1987 proposal for a laser interferometer prototype



MAX-PLANCK-INSTITUT FÜR QUANTENOPTIK

Proposal for the Construction of a Large Laser Interferometer for the Measurement of Gravitational Waves

Translation of Summaries of the Report M P Q 129

Vorschlag zum Bau eines großen Laser-Interferometers zur Messung von Gravitationswellen

– Erweiterte Fassung –

Gerd Leuchs, Karl Maischberger, Albrecht Rüdiger
Roland Schilling, Lise Schnupp, Walter Winkler

M P Q 131 — June 1987


Livingston
Hanford

The Nobel Prize in Physics 2017

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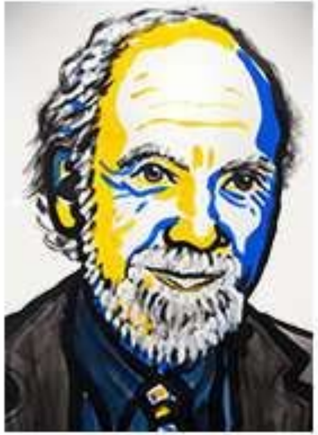
Rainer Weiss

Prize share: 1/2



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Barry C. Barish

Prize share: 1/4

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Kip S. Thorne

Prize share: 1/4

$$= \left(P_{ac}P_{bd} - \tfrac{1}{2}P_{ab}P_{cd} - \tfrac{1}{2}P_{ab}P_{cd} + \tfrac{1}{2}P_{ab}P_{cd}\right) B_{ab}B_{cd} \quad \odot$$

$$= \left(P_{ac}P_{bd} - \tfrac{1}{2}P_{ab}P_{cd}\right) B_{ab}B_{cd}$$

$$= P_{ac}P_{bd}B_{ab}B_{cd} - \tfrac{1}{2}(P_{ab}B_{ab})(P_{cd}B_{cd})$$

$$= \underbrace{P_{ac}P_{bd}B_{ab}B_{cd}} - \tfrac{1}{2}(-B_{ab}n_a n_b)(-B_{cd}n_c n_d)$$

$$(\delta_{ac} - n_a n_c)(\delta_{bd} - n_b n_d)B_{ab}B_{cd}$$

$$= (\delta_{ac}\delta_{bd} + n_a n_b n_c n_d)B_{ab}B_{cd}$$

$$\Rightarrow B_{ab}B_{ab} - 2B_{ac}B_{bc}n_a n_b + \tfrac{1}{2}B_{ab}B_{cd}n_a n_b n_c n_d$$

Since this holds for any symmetric trace free tensor,

and using the given relations, we have that

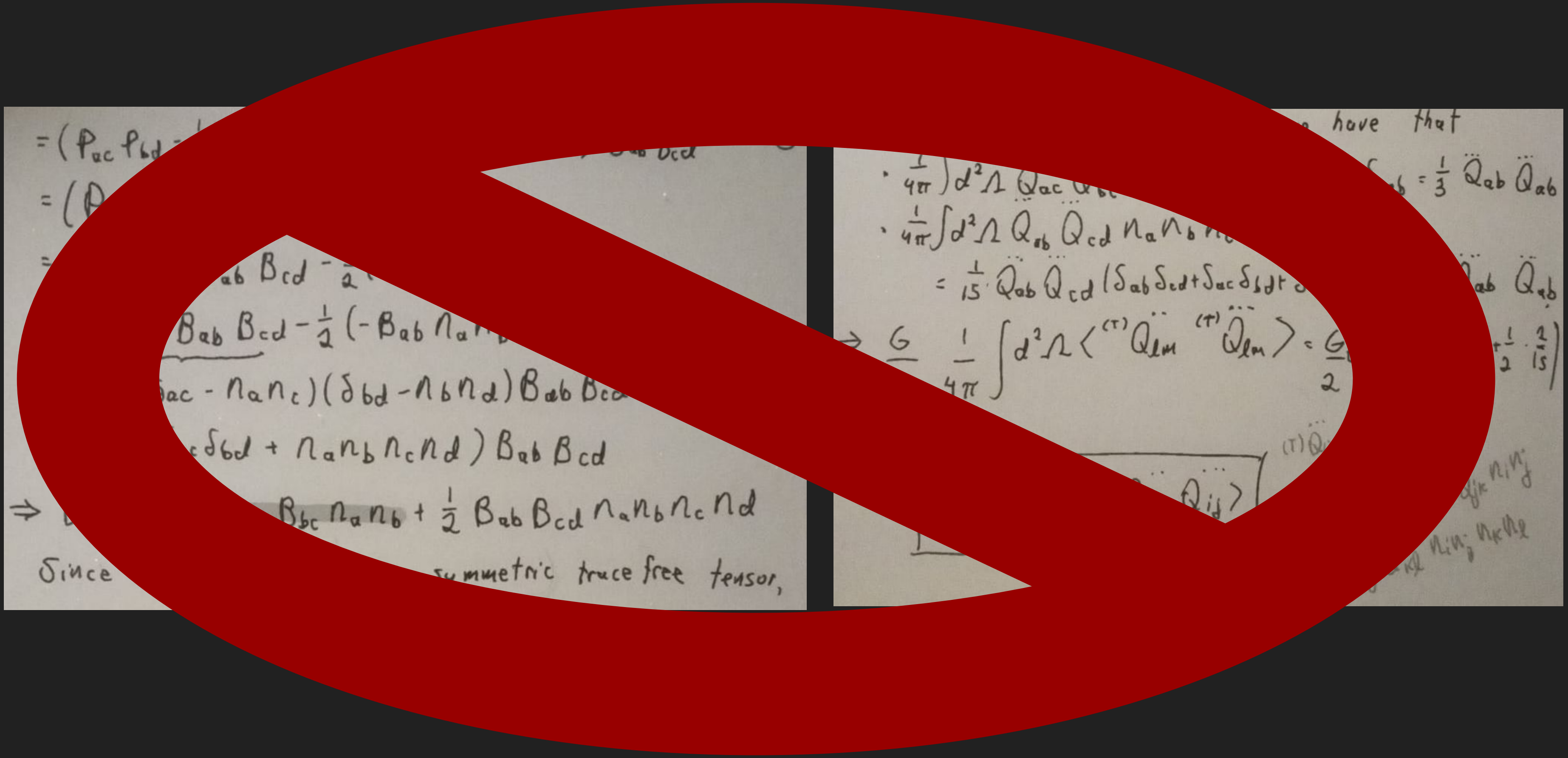
- $\frac{1}{4\pi}\int d^2\Omega\, \dddot{Q}_{ac}\dddot{Q}_{bc}n_a n_b = \frac{1}{3}\dddot{Q}_{ac}\dddot{Q}_{bc}\delta_{ab} = \frac{1}{3}\dddot{Q}_{ab}\dddot{Q}_{ab}$
- $\frac{1}{4\pi}\int d^2\Omega\, \dddot{Q}_{ab}\dddot{Q}_{cd}n_a n_b n_c n_d$

$$= \frac{1}{15}\dddot{Q}_{ab}\dddot{Q}_{cd}(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) = \frac{2}{15}\dddot{Q}_{ab}\dddot{Q}_{ab}$$

$$\Rightarrow \frac{G}{2}\frac{1}{4\pi}\int d^2\Omega \langle {}^{(T)}\ddot{Q}_{lm}\, {}^{(T)}\dddot{Q}_{lm}\rangle = \frac{G}{2}\langle \dddot{Q}_{ab}\dddot{Q}_{ab}\rangle\left(1 - \frac{2}{3} + \frac{1}{2}\cdot\frac{2}{15}\right)$$

$$\Rightarrow \boxed{\frac{dE}{dt} = -\frac{G}{5}\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\rangle}$$

$${}^{(T)}\dddot{Q}_{ij}\, {}^{(T)}\dddot{Q}_{ij} = \dddot{Q}_{ij}\dddot{Q}_{ij} - 2\dddot{Q}_{ik}\dddot{Q}_{jk}n_i n_j + \frac{1}{2}\dddot{Q}_{ij}\dddot{Q}_{kl}n_i n_j n_k n_l$$



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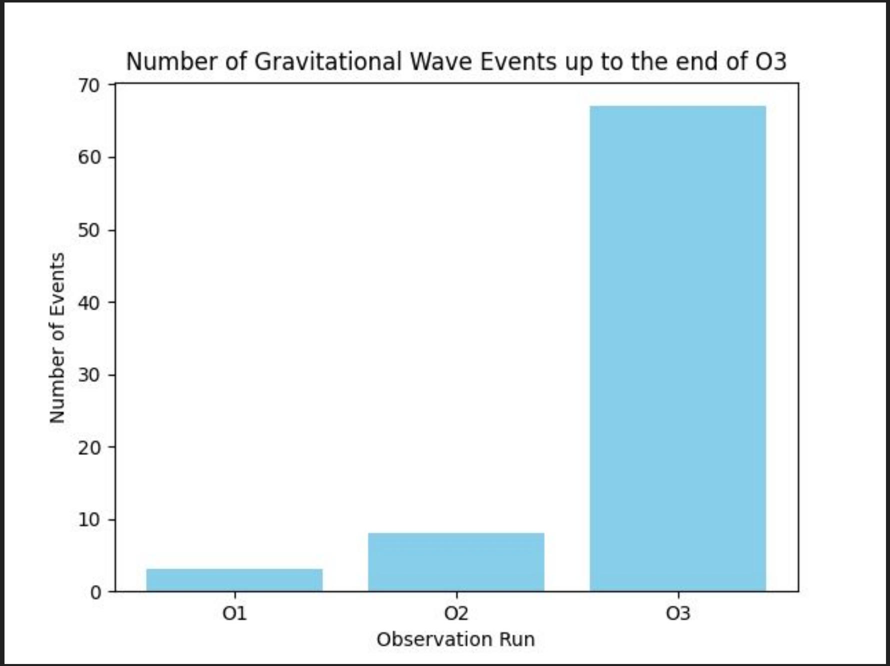
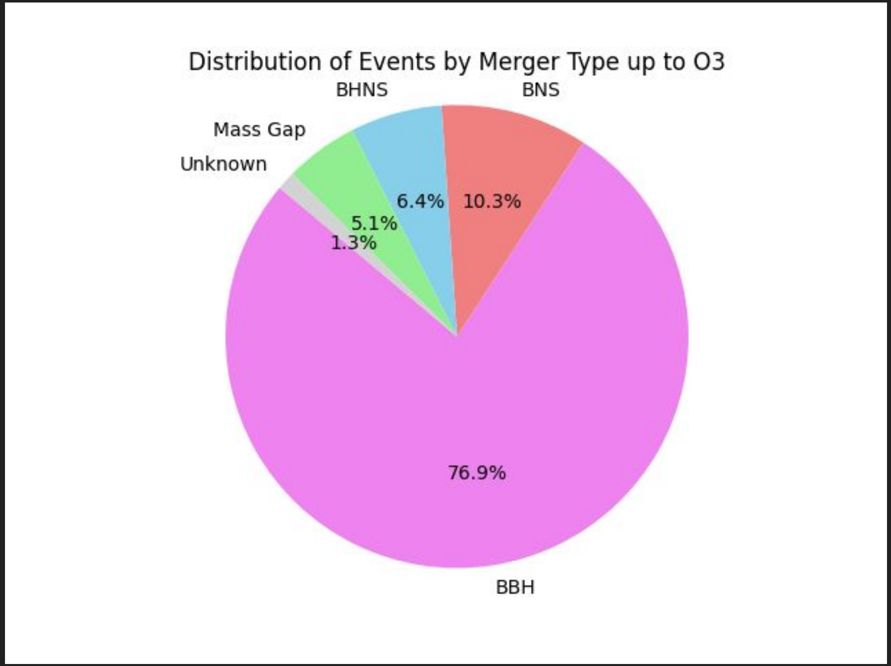
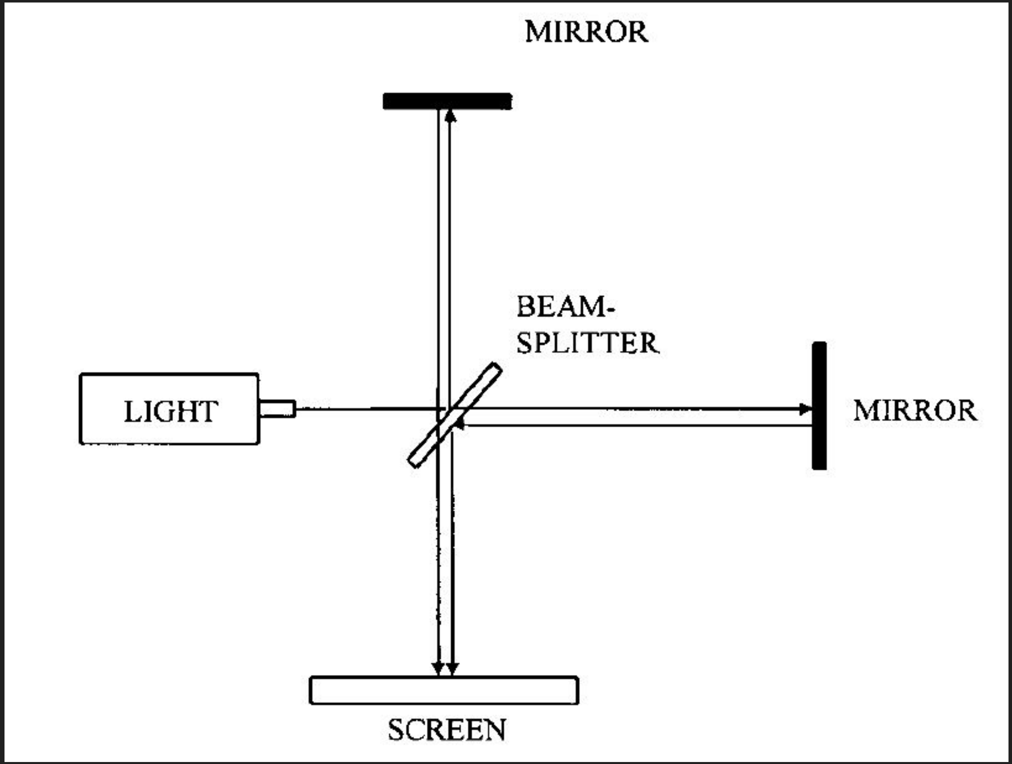




BBH = black hole - black hole merger
BHNS = black hole - neutron star merger
Mass Gap = underdetermined by evidence

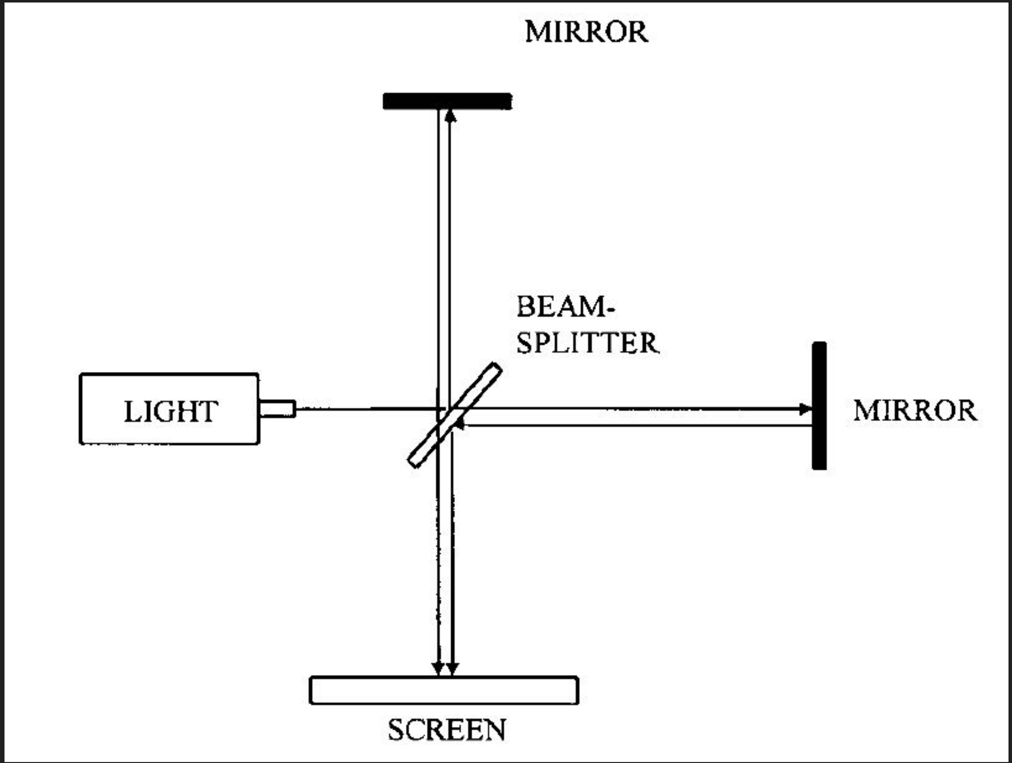
Laser Interferometers

The Big Picture



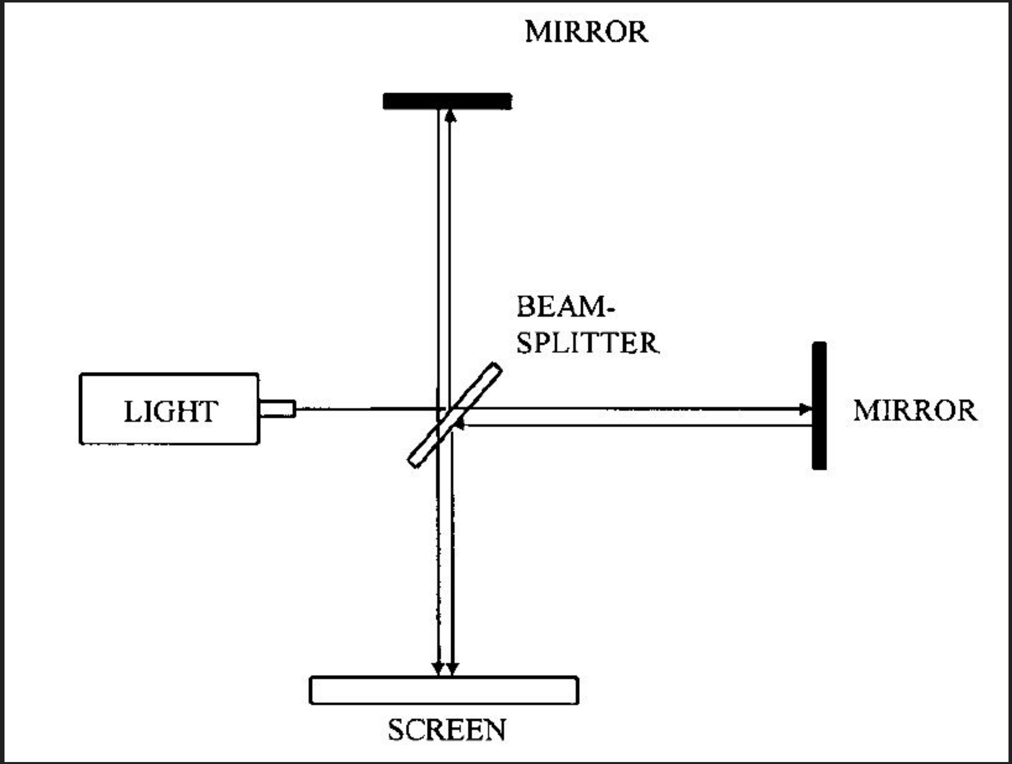
- Differences in the path length travelled by light along the interferometer arms produce an interference pattern

The Big Picture



- Differences in the path length travelled by light along the interferometer arms produce an interference pattern
- Fix the end mirrors to approximate test masses in free fall

The Big Picture



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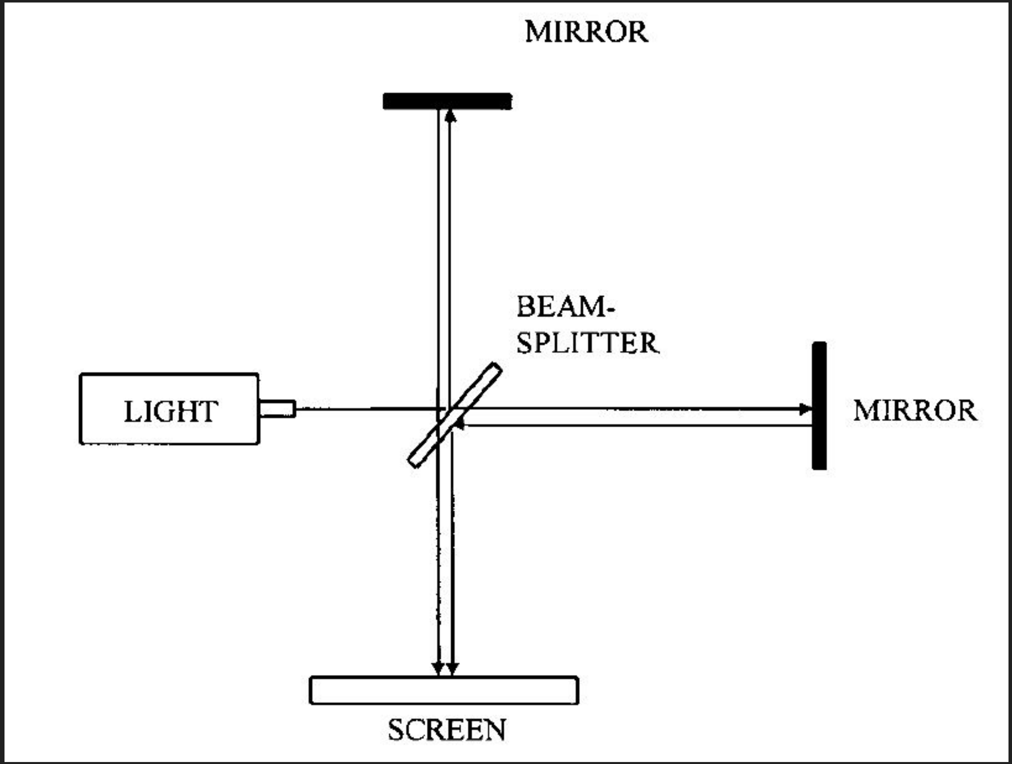
The Big Picture



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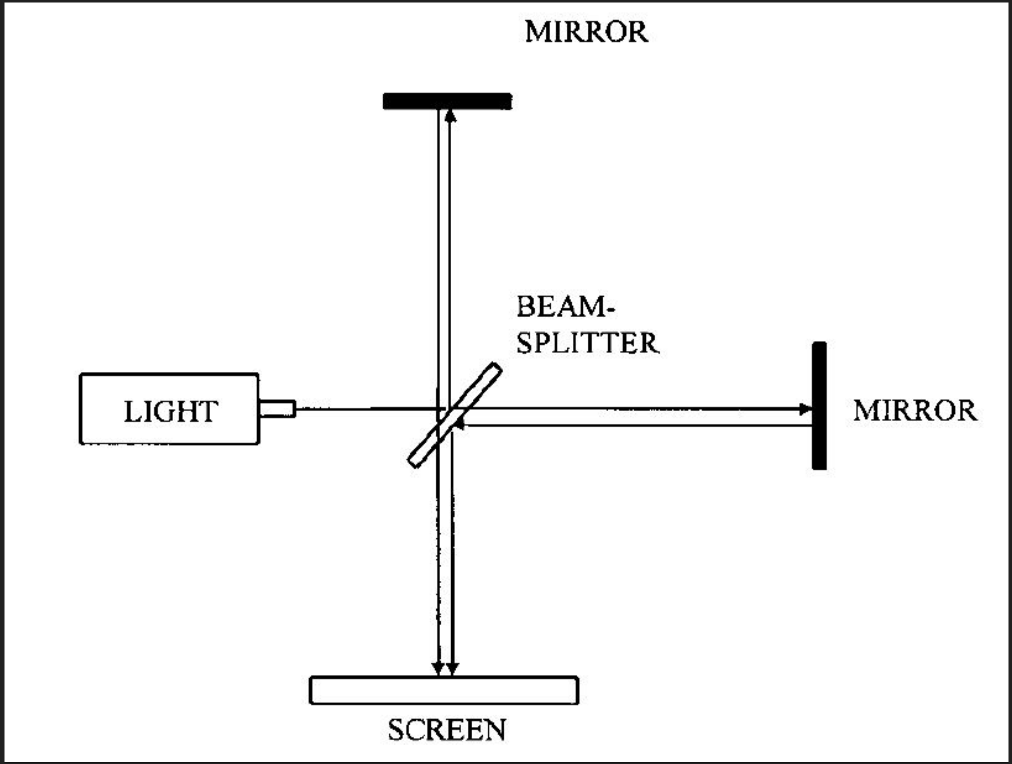
- This produces a **measurable** strain

The Big Picture? Easy enough…



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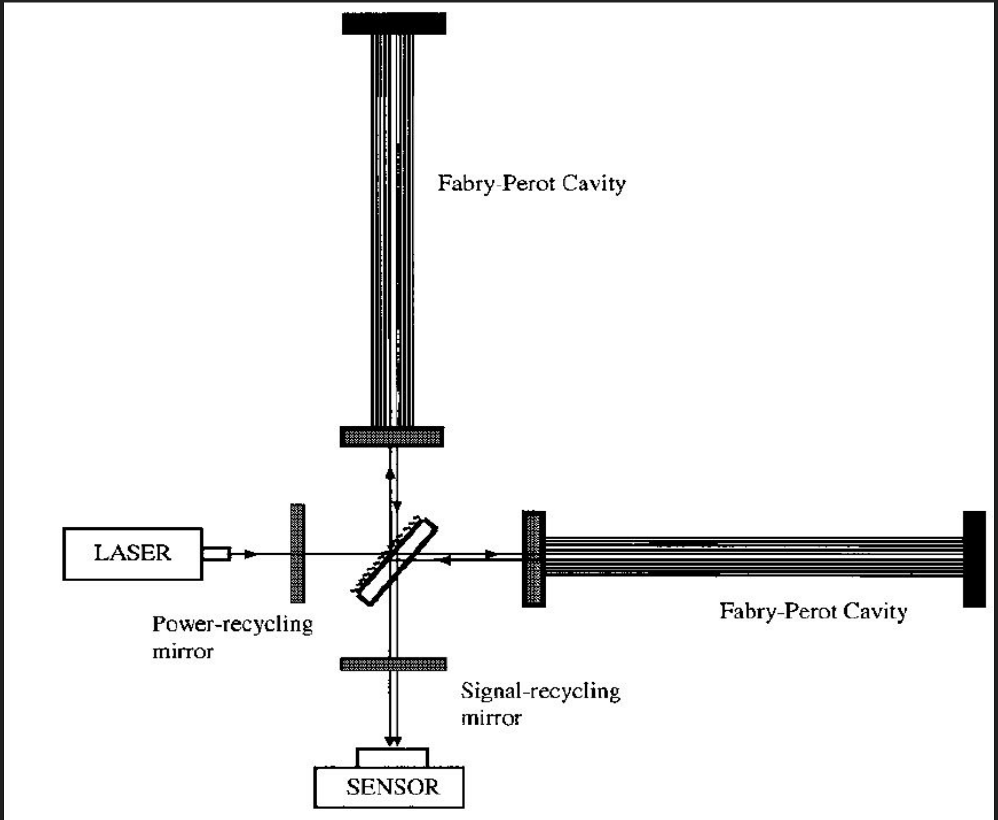
- This produces a **measurable** strain

$$h(t) = \frac{\Delta L_x - \Delta L_y}{L}$$

…but the devil is in the details



- The arms contain Fabry-Pérot cavities to increase the interaction time with potential signals

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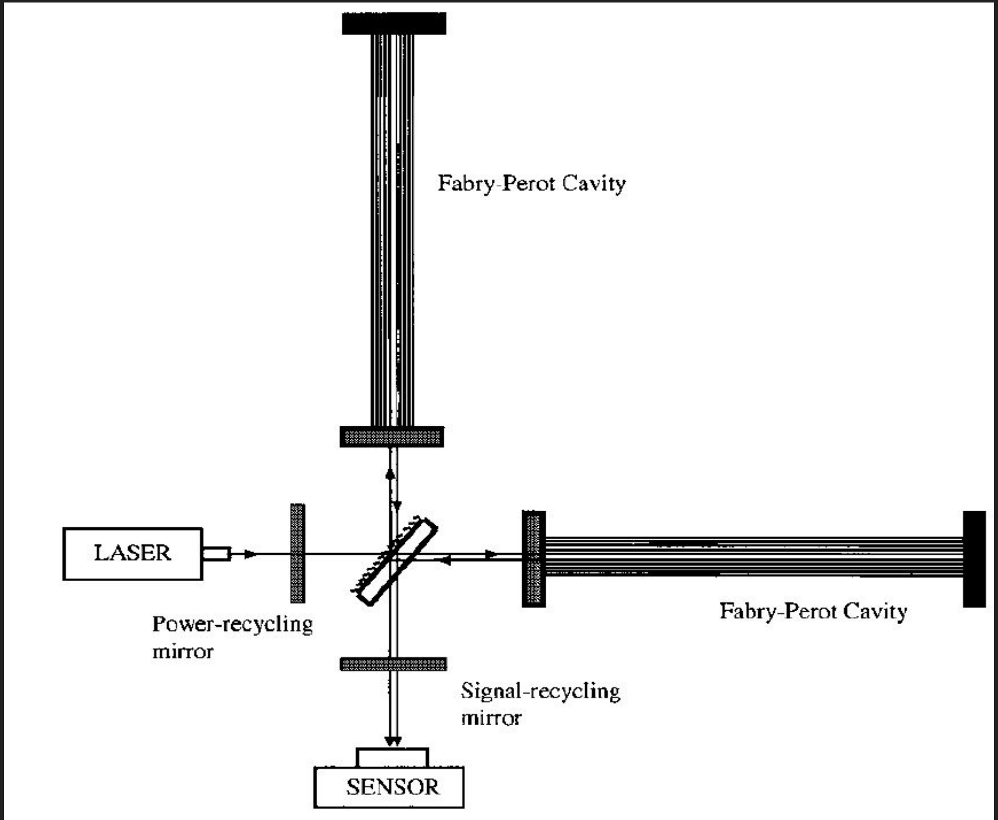

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- Power-recycling mirrors reflect laser light back into the interferometer to increase the power stored

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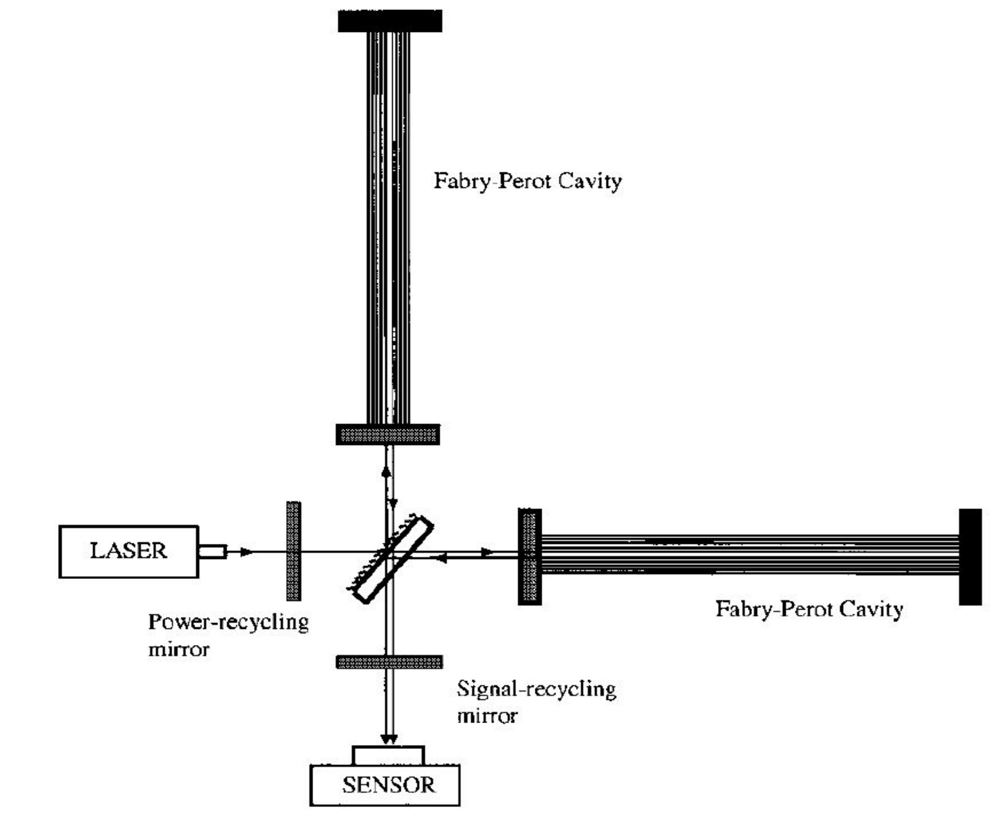
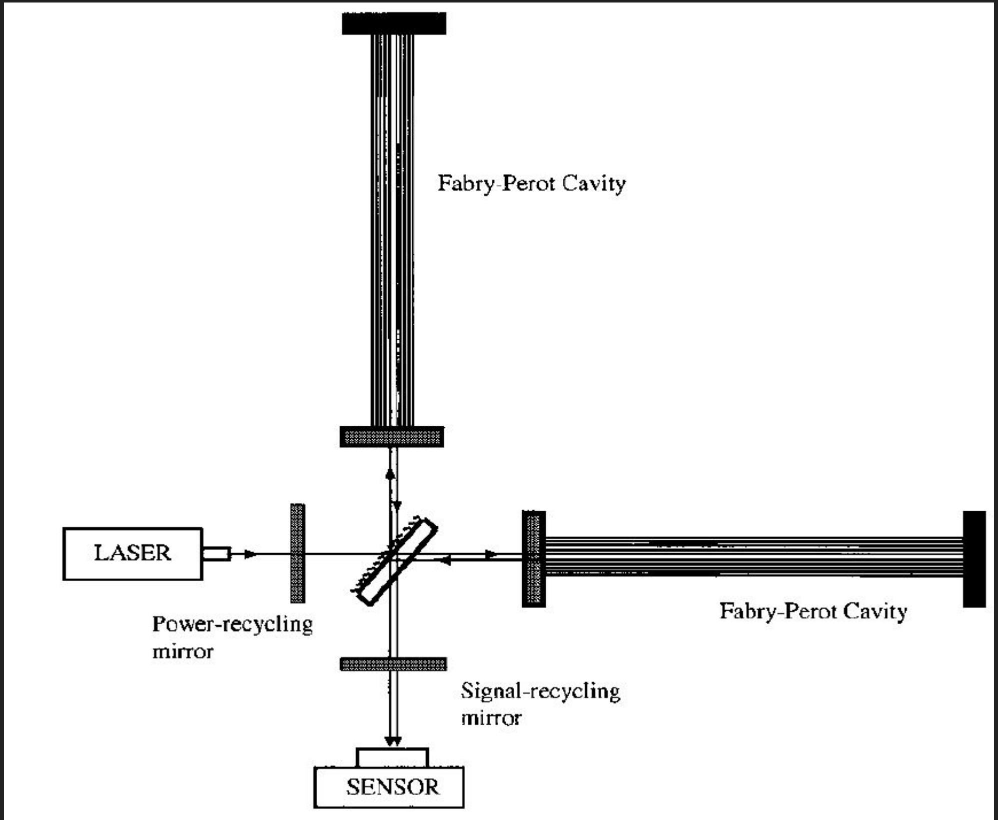


- The arms contain Fabry-Pérot cavities to increase the interaction time with potential signals
- Power-recycling mirrors reflect laser light back into the interferometer to increase the power stored
- Signal-recycling mirrors enhance the interferometer’s sensitivity in the band of interest

…but the devil is in the details



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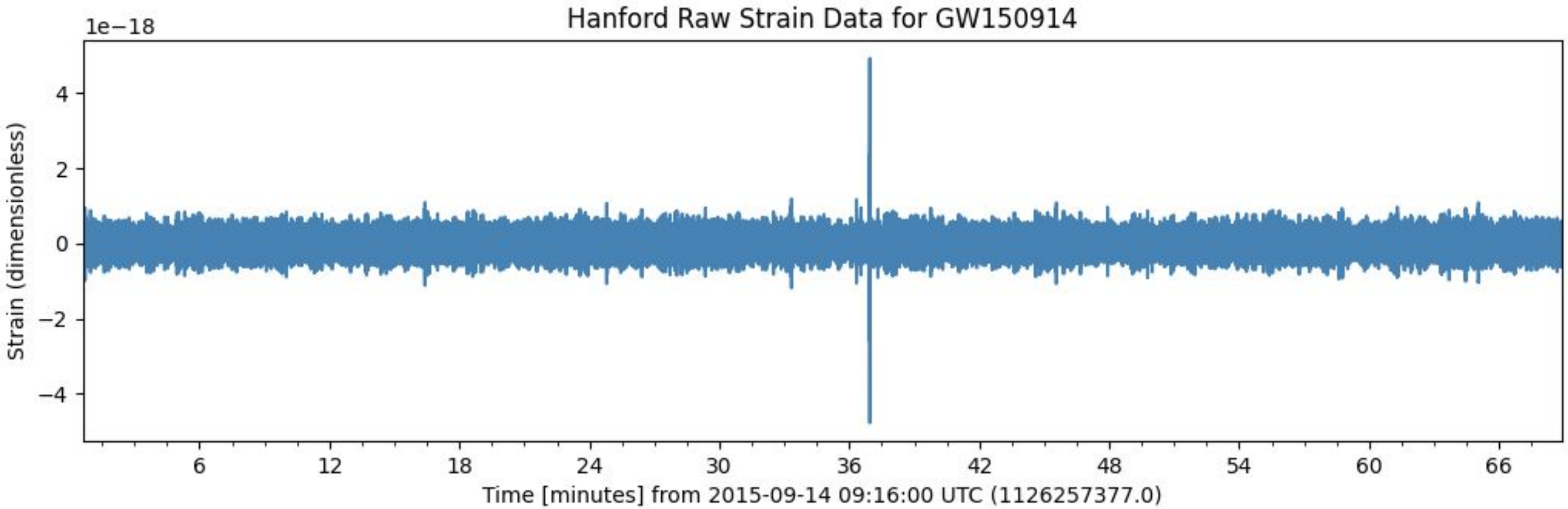
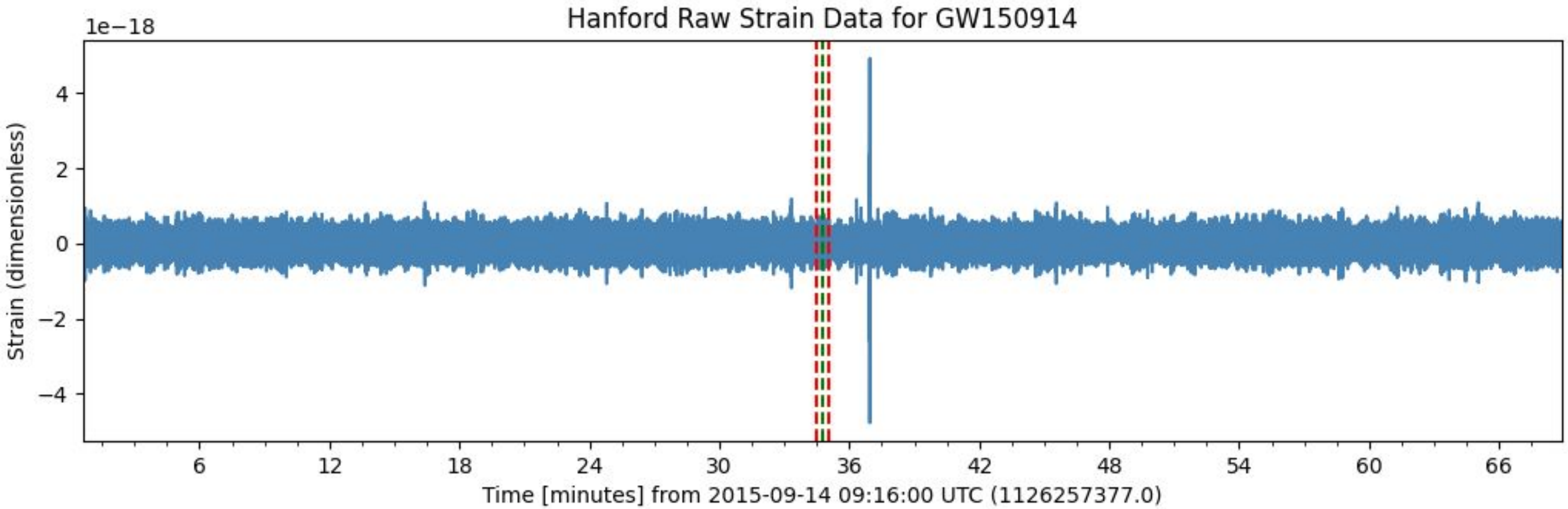
- …

A Look at the Data


Hanford Raw Strain Data for GW150914
1e−18
Strain (dimensionless)
4
2
0
−2
−4
6
12
18
24
30
36
42
48
54
60
66
Time [minutes] from 2015-09-14 09:16:00 UTC (1126257377.0)



Hanford Raw Strain Data for GW150914
1e−18
4
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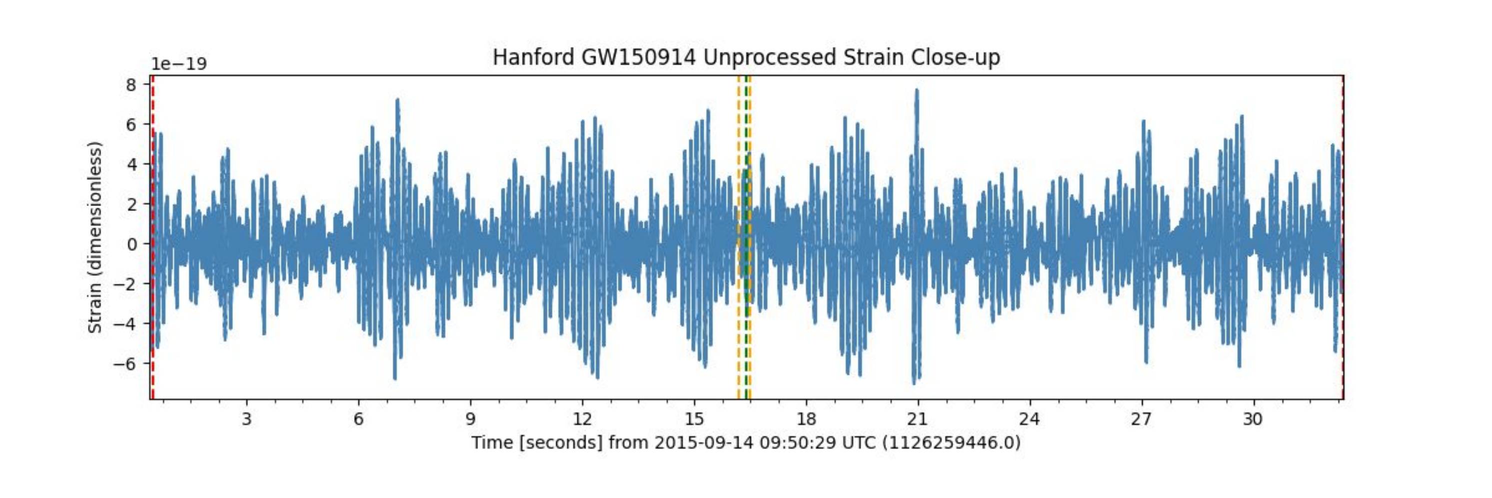
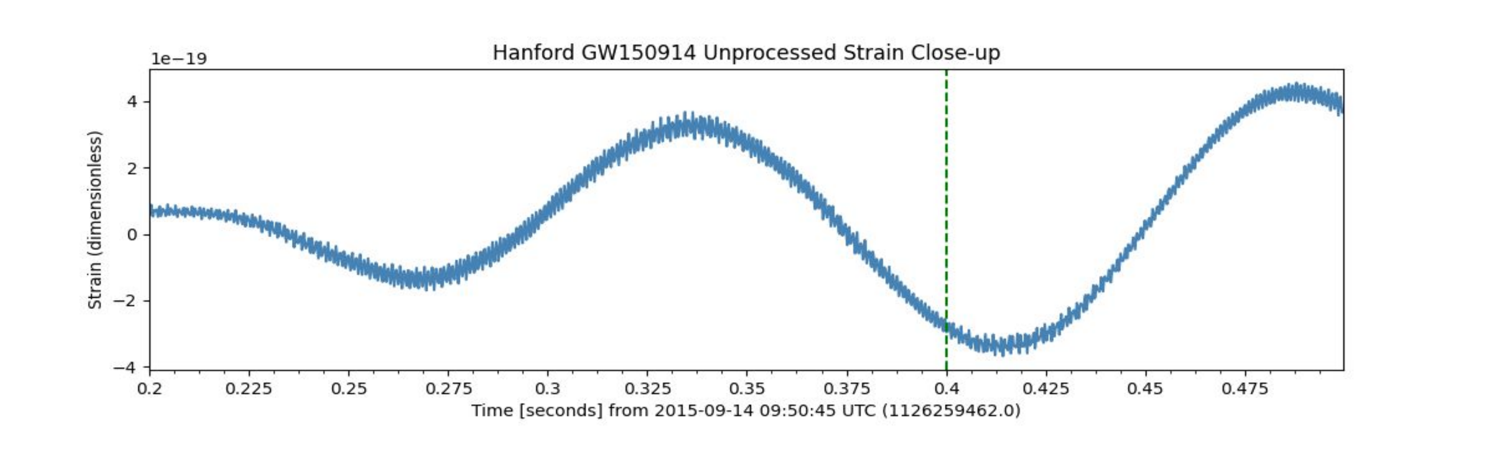
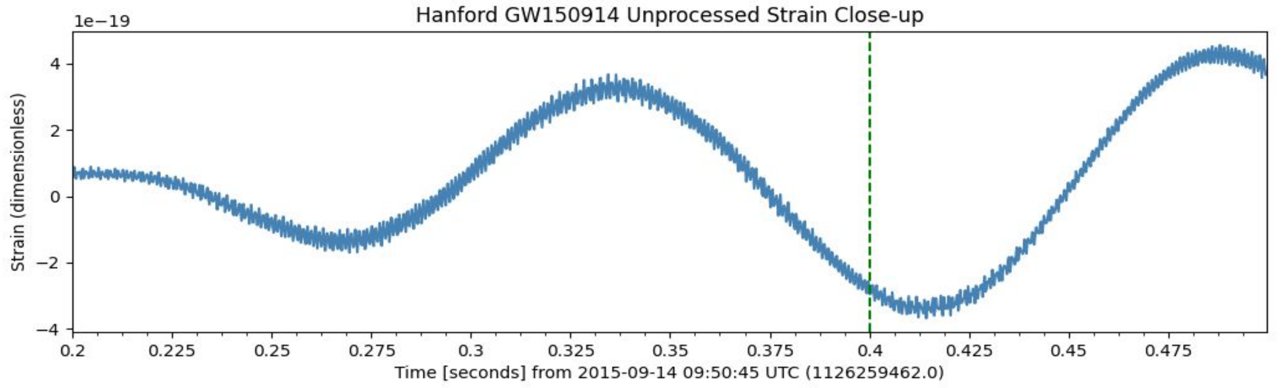
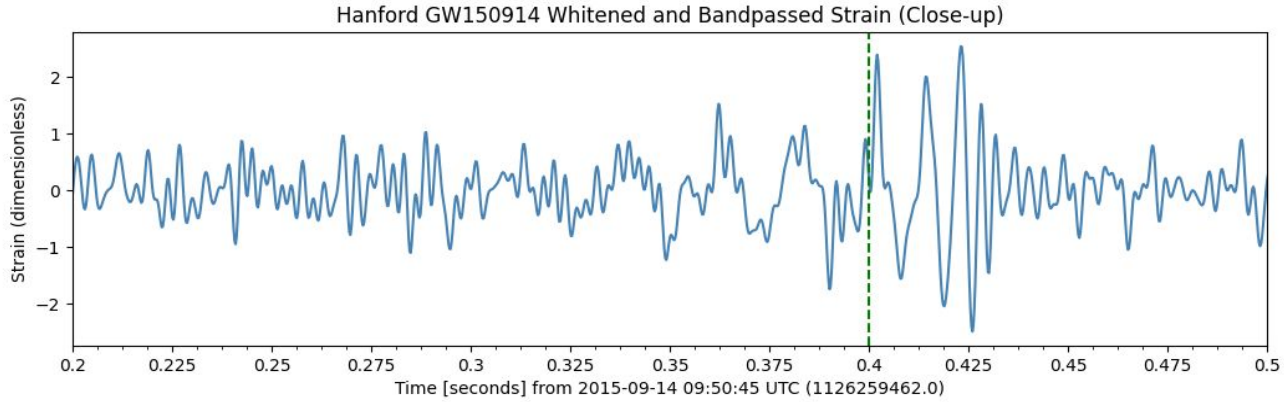
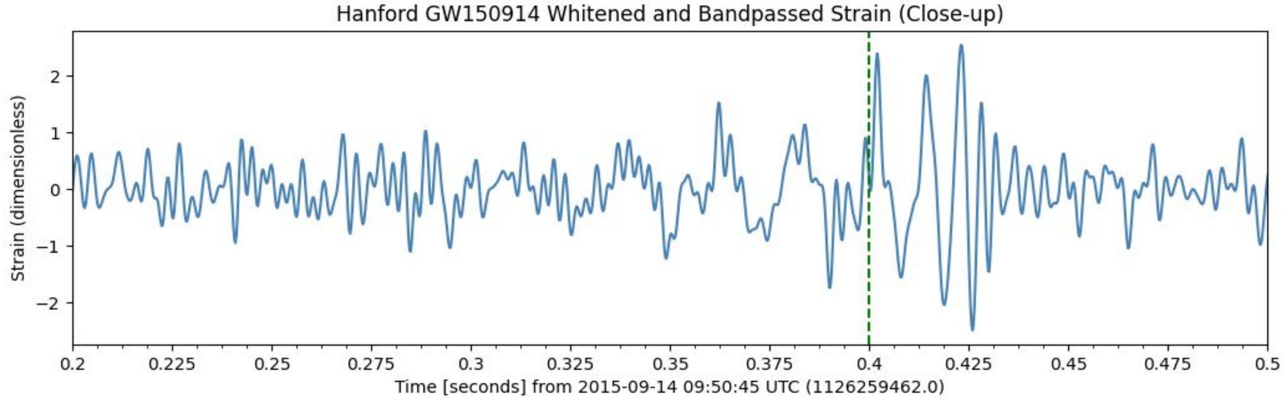
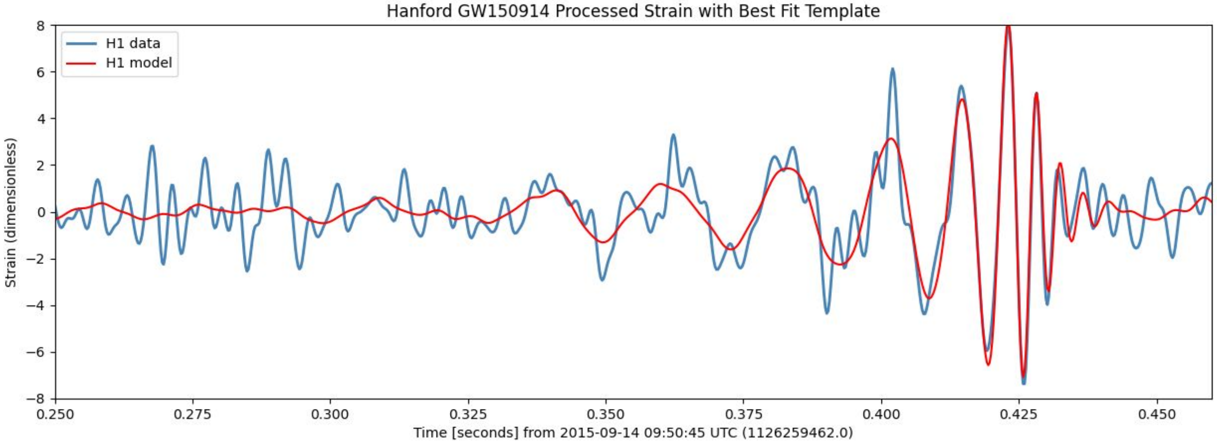

Hanford GW150914 Unprocessed Strain Close-up
1e−19
Strain (dimensionless)
Time [seconds] from 2015-09-14 09:50:29 UTC (1126259446.0)


Hanford GW150914 Unprocessed Strain Close-up
1e−19
Strain (dimensionless)
Time [seconds] from 2015-09-14 09:50:45 UTC (1126259462.0)


Hanford GW150914 Unprocessed Strain Close-up
1e−19
4
2
0
−2
−4
Strain (dimensionless)
0.2
0.225
0.25
0.275
0.3
0.325
0.35
0.375
0.4
0.425
0.45
0.475
Time [seconds] from 2015-09-14 09:50:45 UTC (1126259462.0)

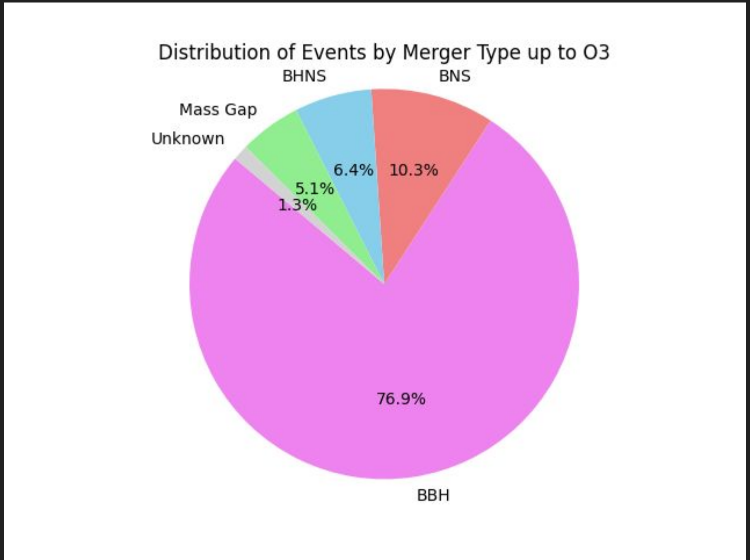

Hanford GW150914 Whitened and Bandpassed Strain (Close-up)
2
1
0
−1
−2
Strain (dimensionless)
0.2
0.225
0.25
0.275
0.3
0.325
0.35
0.375
0.4
0.425
0.45
0.475
0.5
Time [seconds] from 2015-09-14 09:50:45 UTC (1126259462.0)


Hanford GW150914 Whitened and Bandpassed Strain (Close-up)
Strain (dimensionless)
Time [seconds] from 2015-09-14 09:50:45 UTC (1126259462.0)


Hanford GW150914 Processed Strain with Best Fit Template
H1 data
H1 model
Strain (dimensionless)
Time [seconds] from 2015-09-14 09:50:45 UTC (1126259462.0)

So what’s the problem?

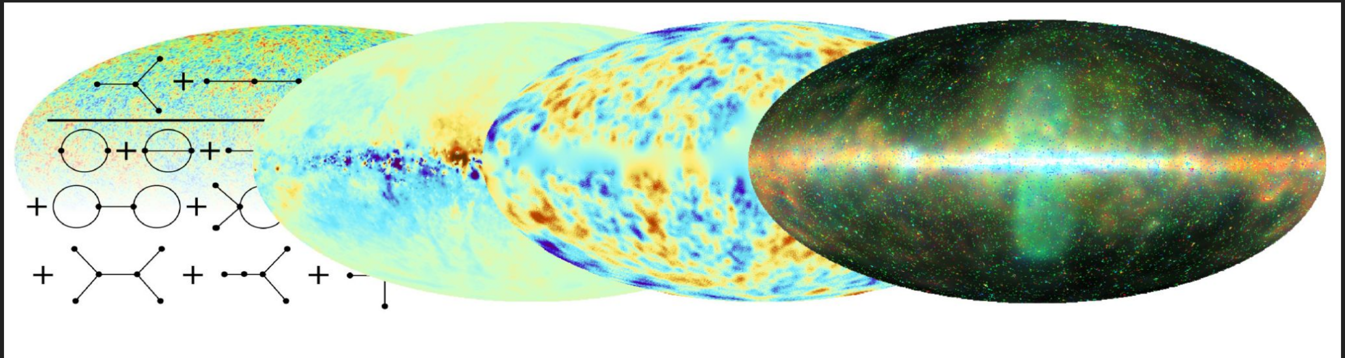
Stellar Binaries	10^{-12} Hz to 10 μHz
Super Massive Black Hole Binaries	10 nHz to 10 mHz
Star-Exoplanet Binaries (ultra-short orbital period)	1 μHz to 1 mHz
Stellar Mass Black Hole Binaries	10 μHz to ~1 kHz
White Dwarf Binaries	1 mHz to 1 Hz
Black Hole–Neutron Star Binaries	10 mHz to 10 kHz
Neutron Star Binaries	100 mHz to ~1 kHz



So what’s the problem?

Can we find a gravitational wave signal in the calibrated strain data *without* relying on any numerical relativity templates?

Information Field Theory



IFT in Brief

- IFT is a Bayesian inference framework that recovers signal fields from noisy, incomplete, or otherwise corrupted measurements

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$$P(s|d) = \frac{P(d|s)}{P(d)} P(s)$$

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- By analogy to statistical field theory, IFT recasts the Bayesian inference posterior as the **information Hamiltonian** to set up optimization problems

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$$P(s|d) = \frac{P(d|s)}{P(d)} P(s)$$

$$H(d,s) = -\ln[P(d,s)]$$

IFT in Brief

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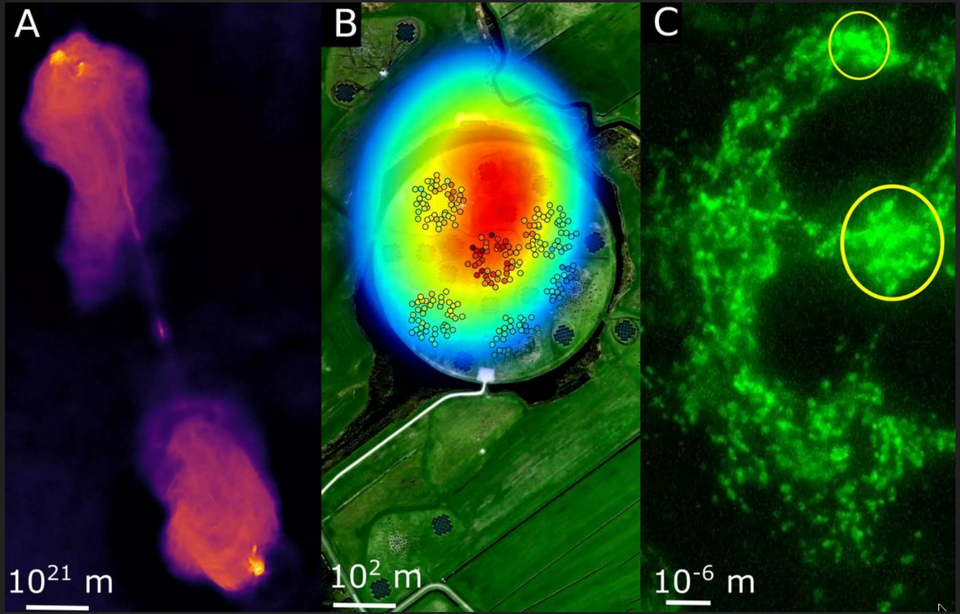
$$P(s|d) = \frac{e^{-H(d,s)}}{Z(d)}$$

$$H(d, s) = -\ln[P(d, s)]$$

IFT in Brief

- IFT is a Bayesian inference framework that recovers signal fields from noise, incomplete, or otherwise corrupted measurements
- By analogy to statistical field theory, IFT recasts the Bayesian inference posterior as the **information Hamiltonian** to set up optimization problems
- Proven successes:
  - analysis of the CMB
  - X-ray and radio tomography
  - medical imaging



The Measurement Equation

$$d = Rs + n$$

The Measurement Equation

$$d = Rs + n$$

data

The Measurement Equation

signal

$$d = Rs + n$$

data

The Measurement Equation

signal

$$d = Rs + n$$

data

noise

The Measurement Equation

Response signal

$$d = Rs + n$$

data noise

The Likelihood

$$P(d|s) = G(d - Rs, N)$$

The Likelihood

$$P(d|s) = G(d - Rs, N)$$

Noise
Covariance

The Likelihood

$$P(d|s) = G(d - Rs, N)$$

Noise Covariance

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Noise Covariance

What about the Prior?

- Bayesian inference requires an initial guess of the prior, $P(s)$

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Operator derived from hyperparameters

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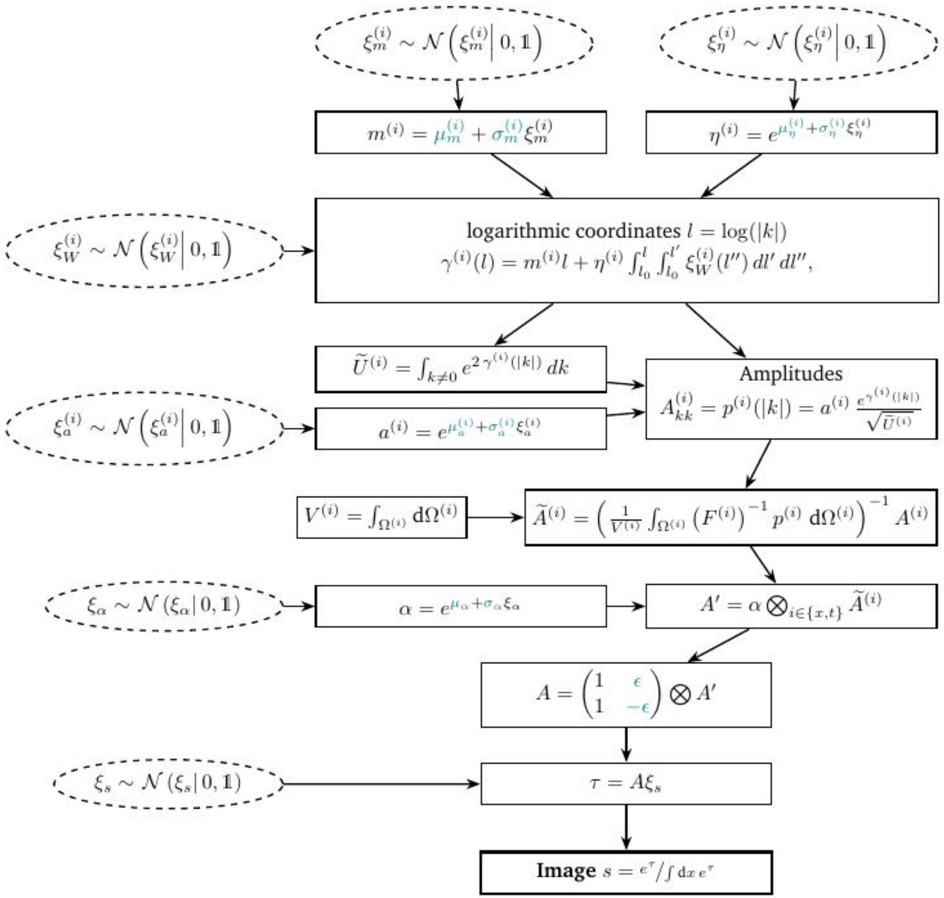
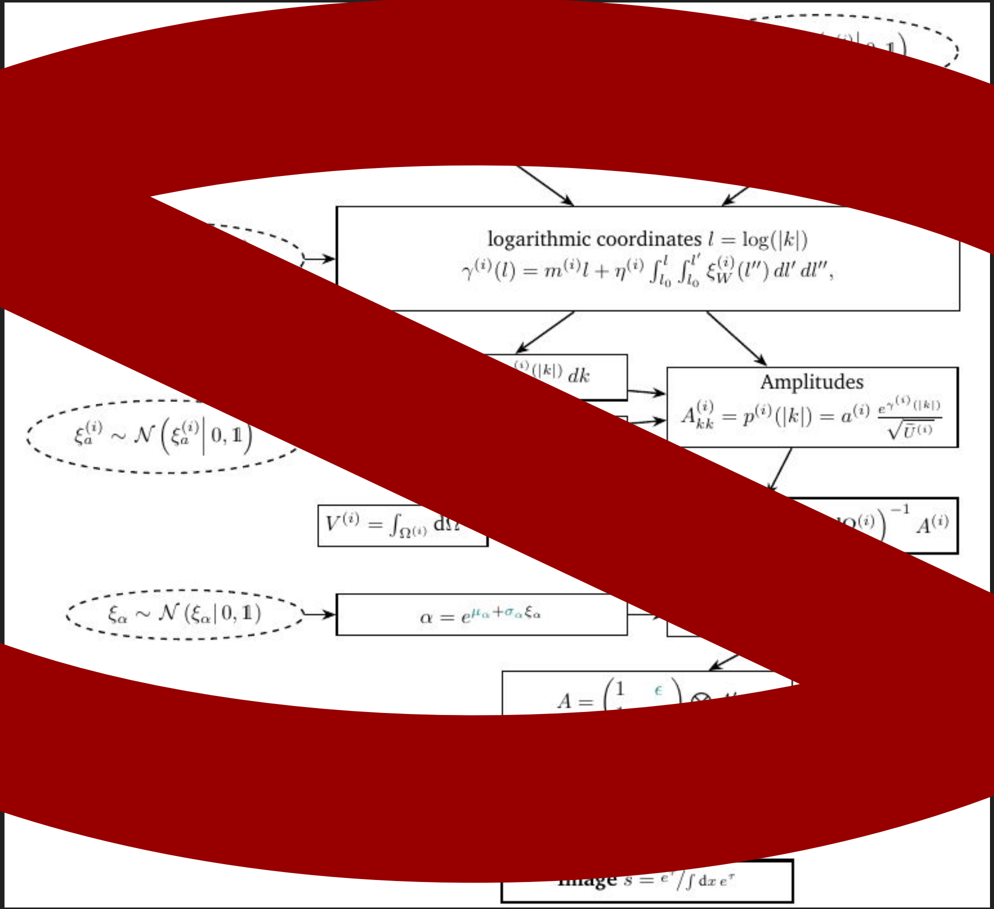
Operator derived from hyperparameters



$\xi_m^{(i)} \sim \mathcal{N}\left(\xi_m^{(i)} \middle| 0, \mathbb{1}\right)$
$\xi_\eta^{(i)} \sim \mathcal{N}\left(\xi_\eta^{(i)} \middle| 0, \mathbb{1}\right)$
$m^{(i)} = \mu_m^{(i)} + \sigma_m^{(i)} \xi_m^{(i)}$
$\eta^{(i)} = e^{\mu_\eta^{(i)} + \sigma_\eta^{(i)} \xi_\eta^{(i)}}$
$\xi_W^{(i)} \sim \mathcal{N}\left(\xi_W^{(i)} \middle| 0, \mathbb{1}\right)$
logarithmic coordinates $l = \log(|k|)$
$\gamma^{(i)}(l) = m^{(i)} l + \eta^{(i)} \int_{l_0}^{l} \int_{l_0}^{l'} \xi_W^{(i)}(l'')\, dl'\, dl''$,
$\widetilde{U}^{(i)} = \int_{k \neq 0} e^{2\gamma^{(i)}(|k|)}\, dk$
Amplitudes
$A_{kk}^{(i)} = p^{(i)}(|k|) = a^{(i)} \frac{e^{\gamma^{(i)}(|k|)}}{\sqrt{\widetilde{U}^{(i)}}}$
$\xi_a^{(i)} \sim \mathcal{N}\left(\xi_a^{(i)} \middle| 0, \mathbb{1}\right)$
$a^{(i)} = e^{\mu_a^{(i)} + \sigma_a^{(i)} \xi_a^{(i)}}$
$V^{(i)} = \int_{\Omega^{(i)}} \mathrm{d}\Omega^{(i)}$
$\widetilde{A}^{(i)} = \left(\frac{1}{V^{(i)}} \int_{\Omega^{(i)}} \left(F^{(i)}\right)^{-1} p^{(i)}\, \mathrm{d}\Omega^{(i)} \right)^{-1} A^{(i)}$
$\xi_\alpha \sim \mathcal{N}(\xi_\alpha | 0, \mathbb{1})$
$\alpha = e^{\mu_\alpha + \sigma_\alpha \xi_\alpha}$
$A' = \alpha \bigotimes_{i \in \{x,t\}} \widetilde{A}^{(i)}$
$A = \begin{pmatrix} 1 & \epsilon \\ 1 & -\epsilon \end{pmatrix} \bigotimes A'$
$\xi_s \sim \mathcal{N}(\xi_s | 0, \mathbb{1})$
$\tau = A\xi_s$
Image $s = e^{\tau} / \int \mathrm{d}x\, e^{\tau}$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Amplitudes
$A^{(i)}_{kk} = p^{(i)}(|k|) = a^{(i)} \frac{e^{\gamma^{(i)}(|k|)}}{\sqrt{\tilde{U}^{(i)}}}$
$\xi_a^{(i)} \sim \mathcal{N}\left(\xi_a^{(i)}\middle|\,0, \mathbb{1}\right)$
$\xi_\alpha \sim \mathcal{N}(\xi_\alpha|\,0, \mathbb{1})$
$\alpha = e^{\mu_\alpha+\sigma_\alpha\xi_\alpha}$

NIFTy Implementation

Roadmap to Reconstruction

1. Acquire strain data for a confirmed event

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2. Delineate the time interval in which we expect to find the signal

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5. Define the likelihood energy

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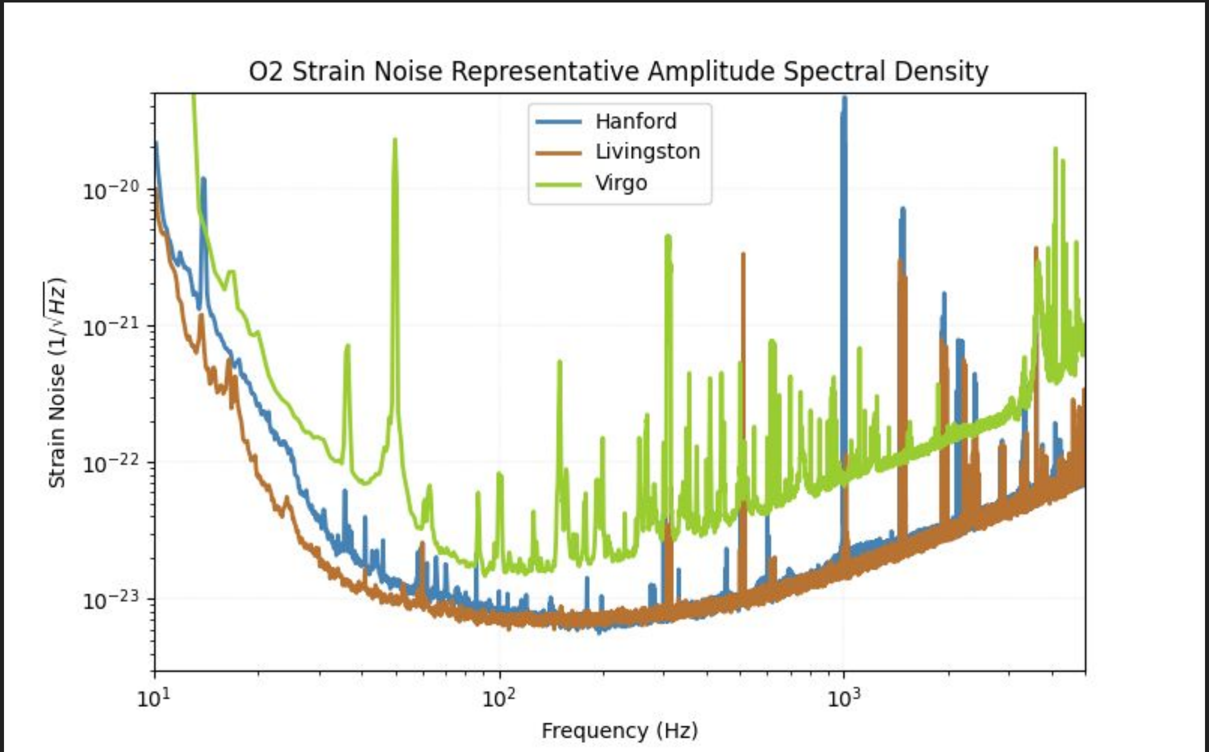
4. Specify the instrument response and noise covariance operators
5. Define the likelihood energy
6. Minimize the Kullback-Leibler divergence thereof
7. Draw samples from the optimization results for comparison with the LVC's published signal

Determining the Operators

Instrument Noise

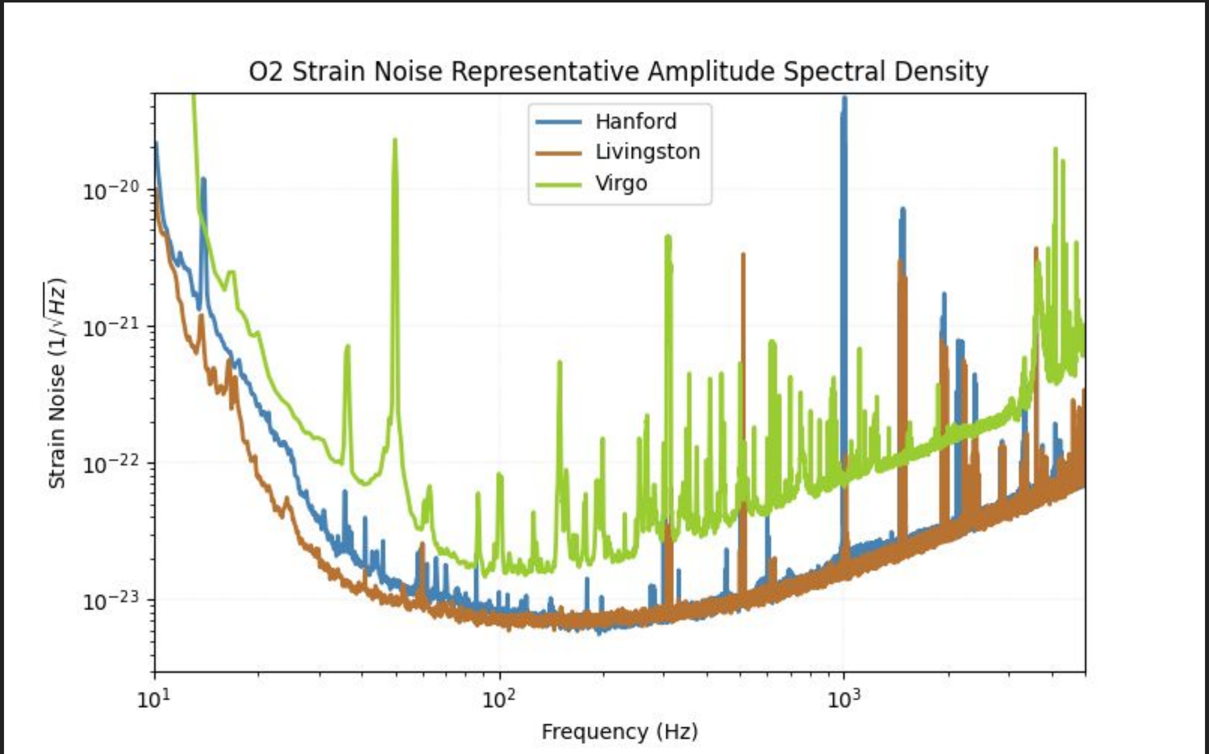
Noise Sources

- Shot noise



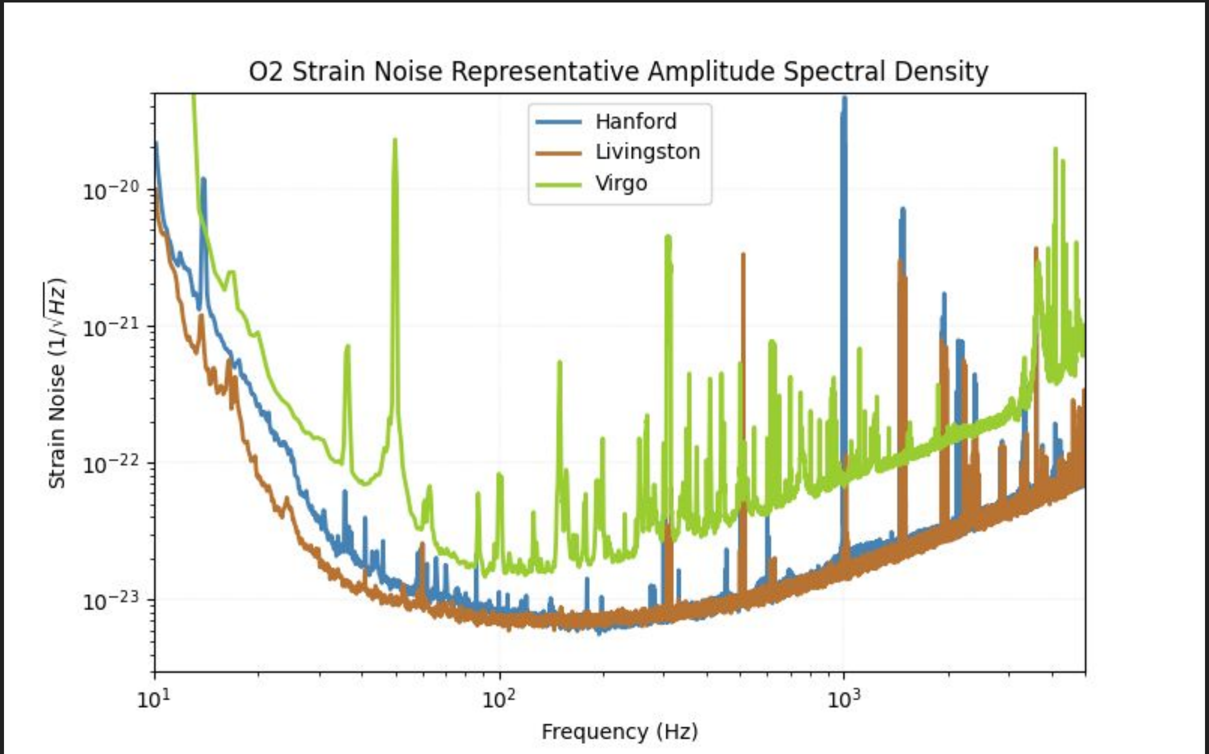
Noise Sources

- Shot noise
- Radiation pressure noise



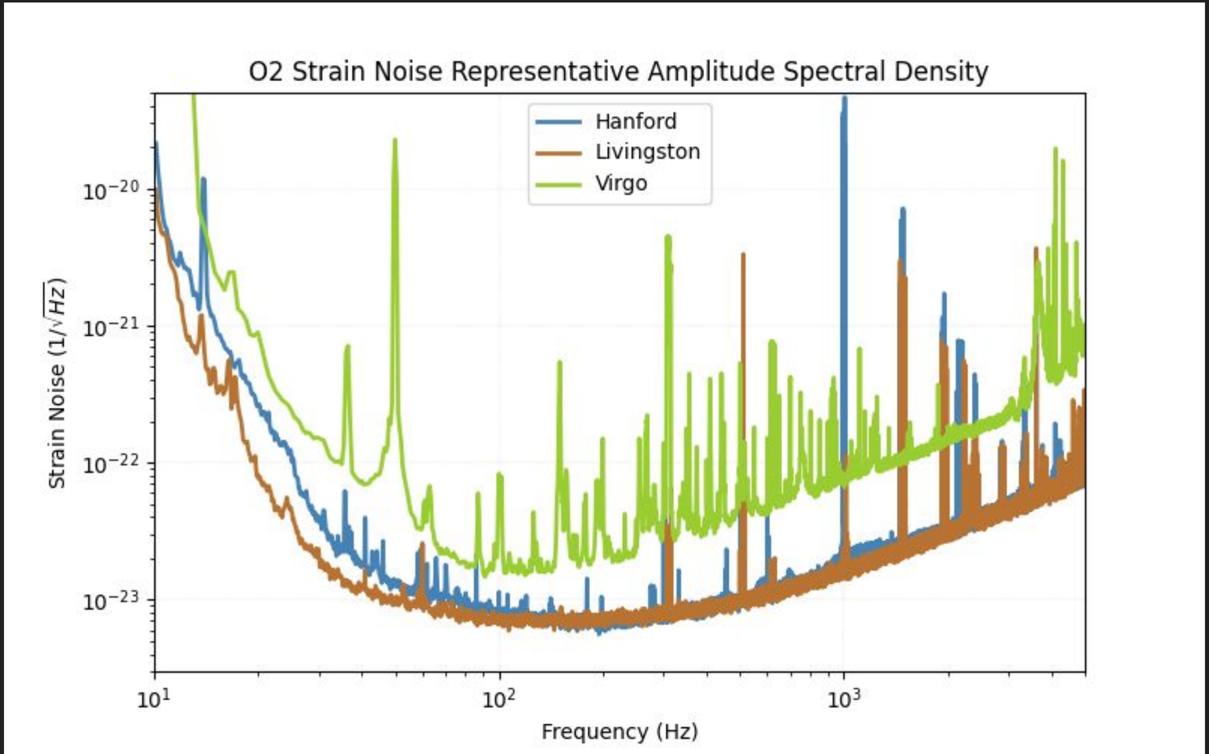
Noise Sources

- Shot noise
- Radiation pressure noise
- Test mass thermal noise



Noise Sources

- Shot noise
- Radiation pressure noise
- Test mass thermal noise
- Coating Brownian noise



Noise Sources

- Shot noise
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- Test mass thermal noise
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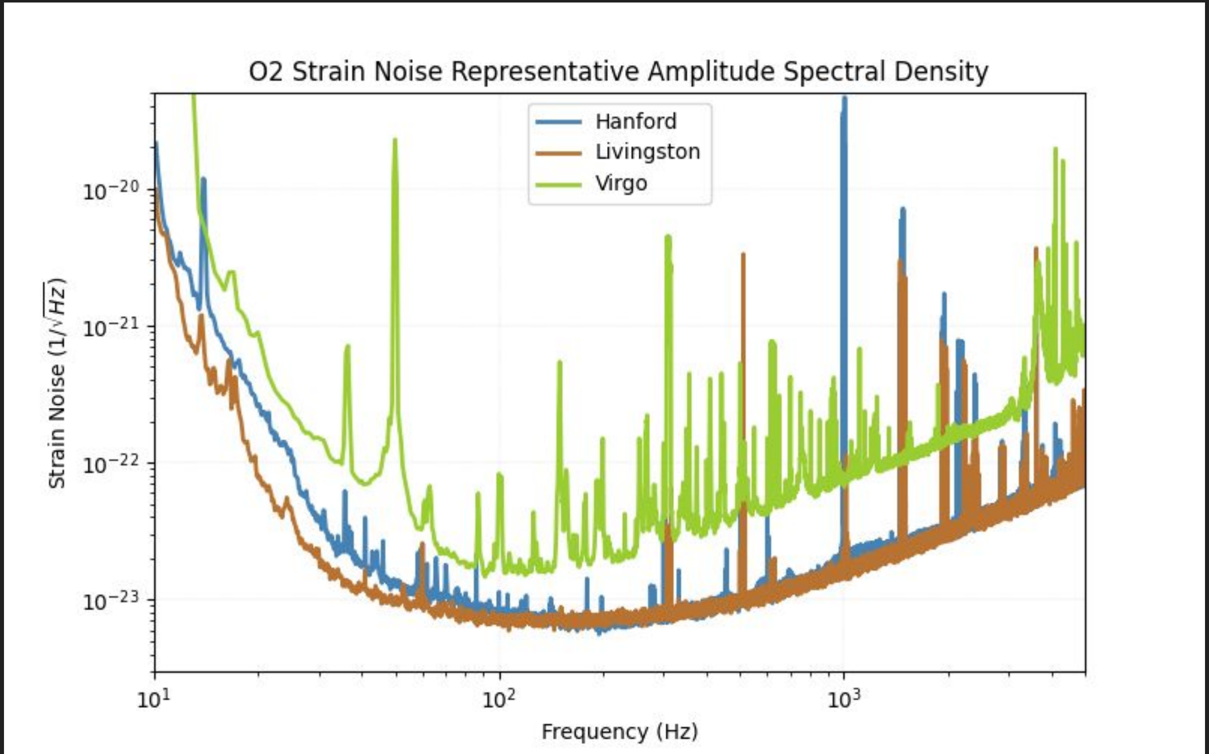
- Thermo-optic noise



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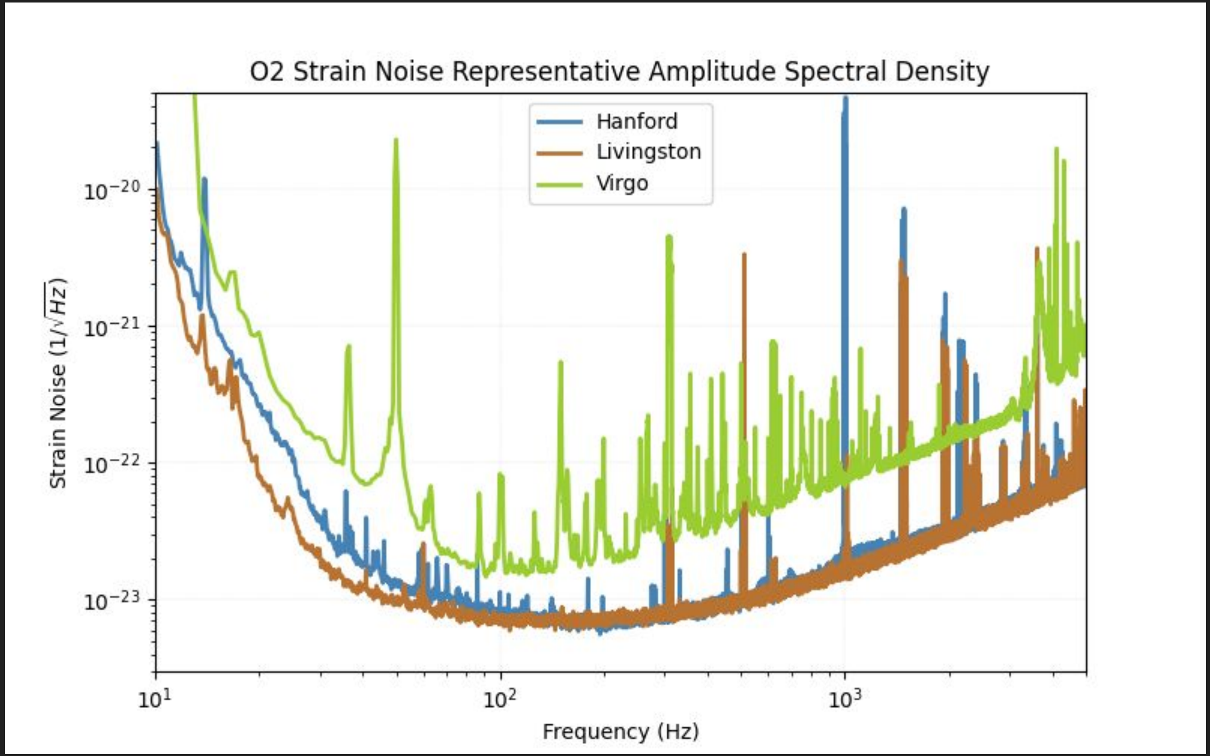
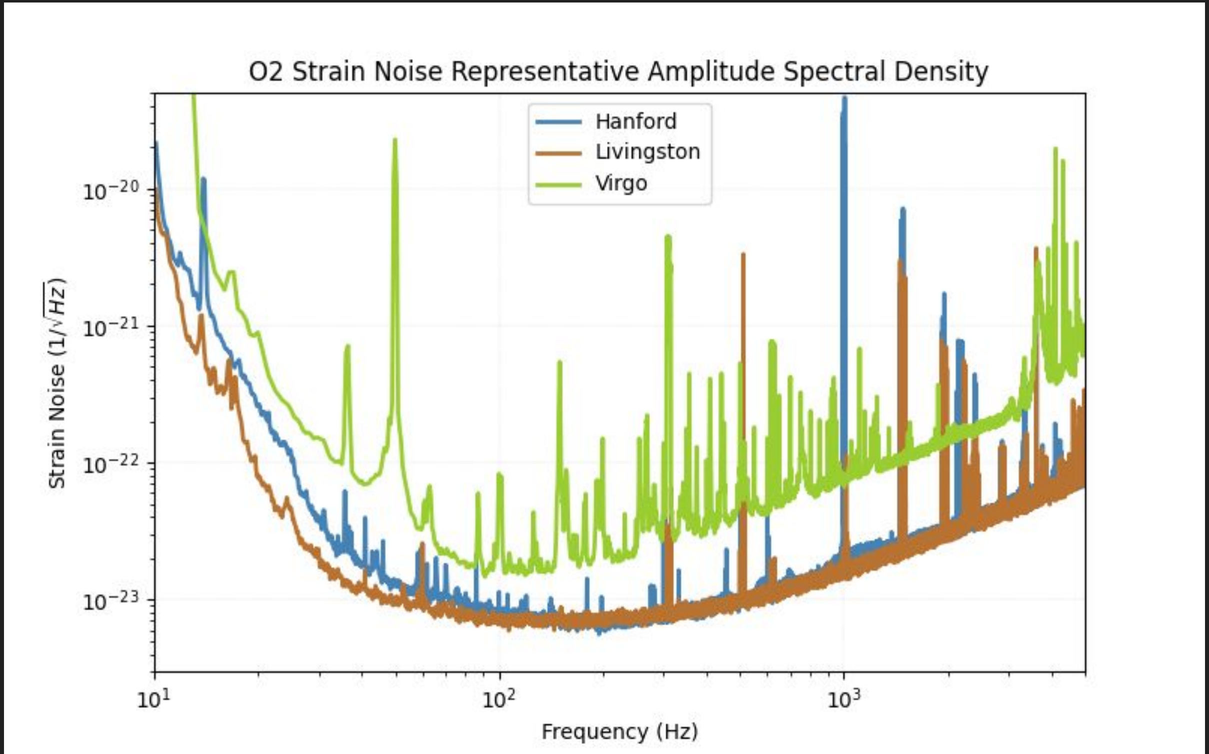


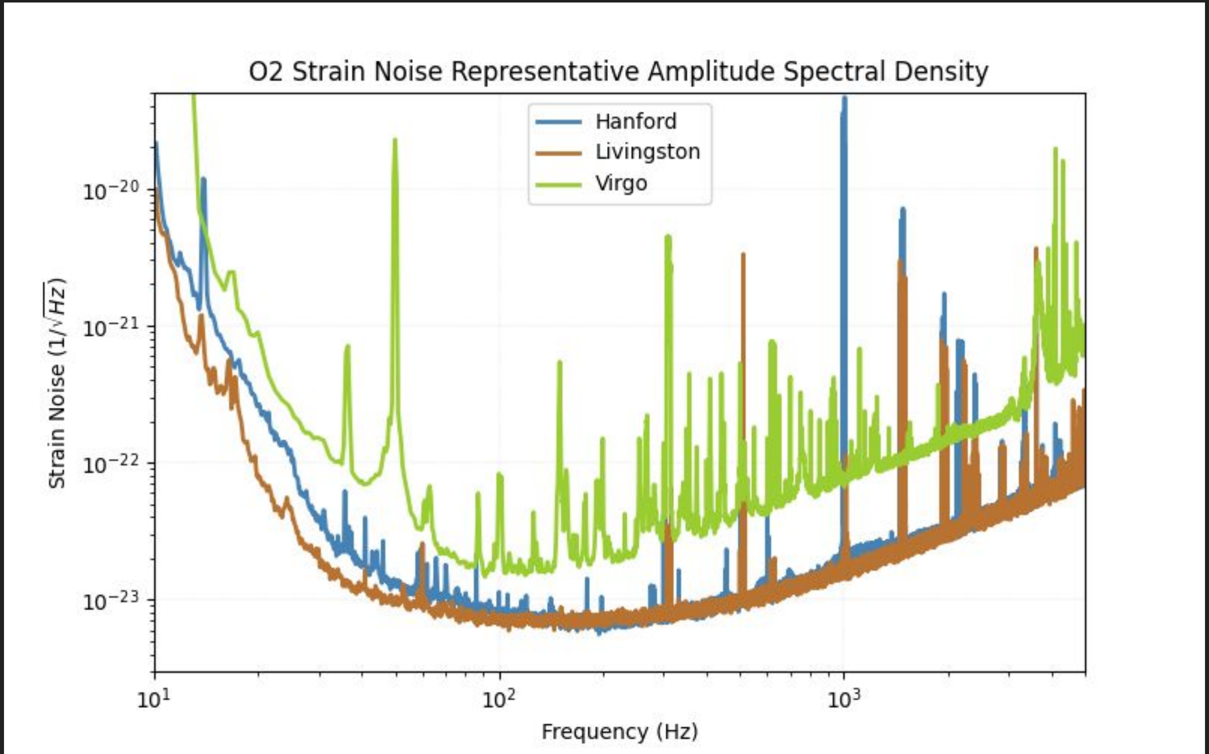
Noise Sources

- Shot noise
- Radiation pressure noise
- Test mass thermal noise
- Coating Brownian noise
- Thermo-optic noise
- Suspension thermal noise
- Gravity gradient noise



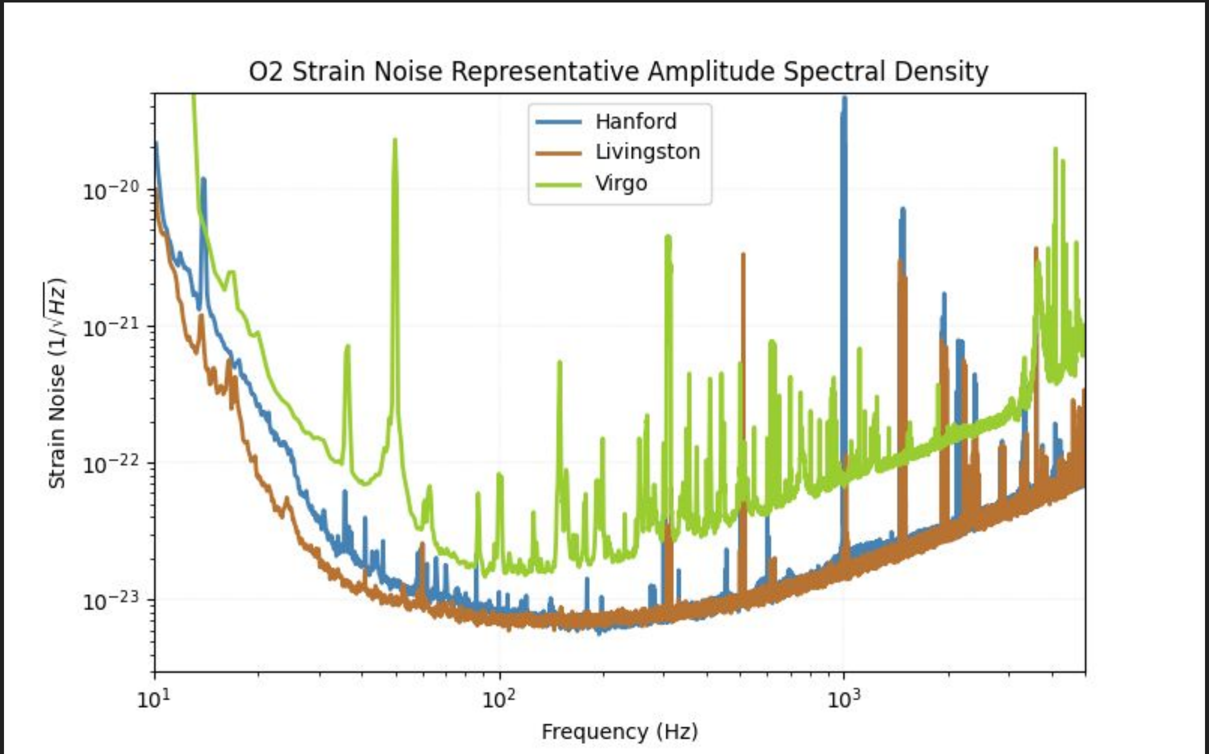
Noise Sources

- Shot noise
- Radiation pressure noise
- Test mass thermal noise
- Coating Brownian noise
- Thermo-optic noise
- Suspension thermal noise
- Gravity gradient noise
- Residual gas noise



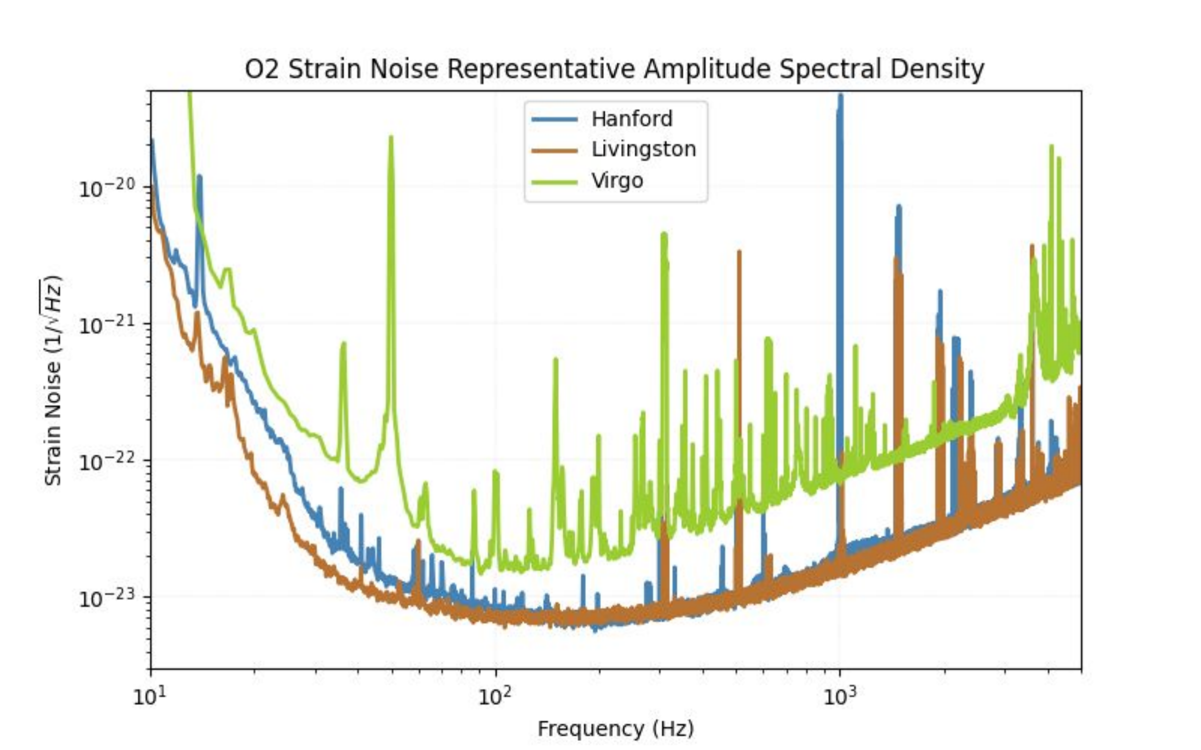
Noise Sources

- Shot noise
- Radiation pressure noise
- Test mass thermal noise
- Coating Brownian noise
- Thermo-optic noise
- Suspension thermal noise
- Gravity gradient noise
- Residual gas noise
- …



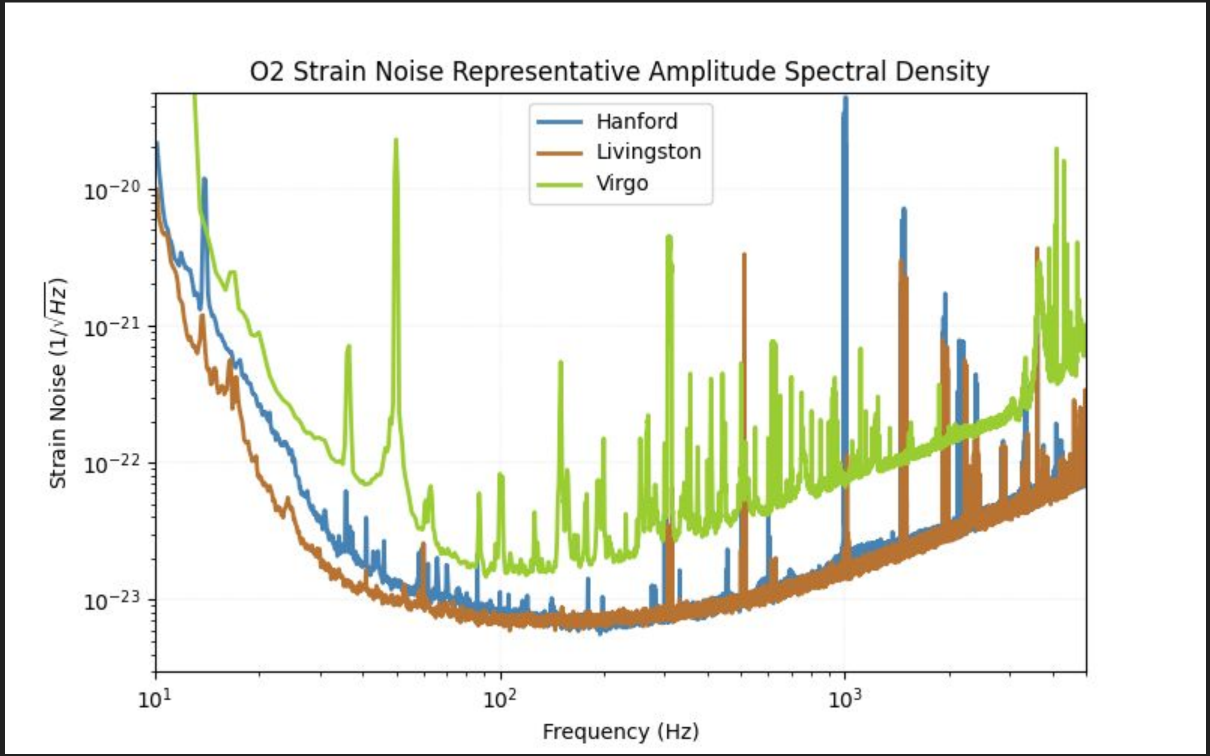
Modelling the Noise

- For a given run, the LVC chooses a **quiet** time series interval during the interferometer's operation



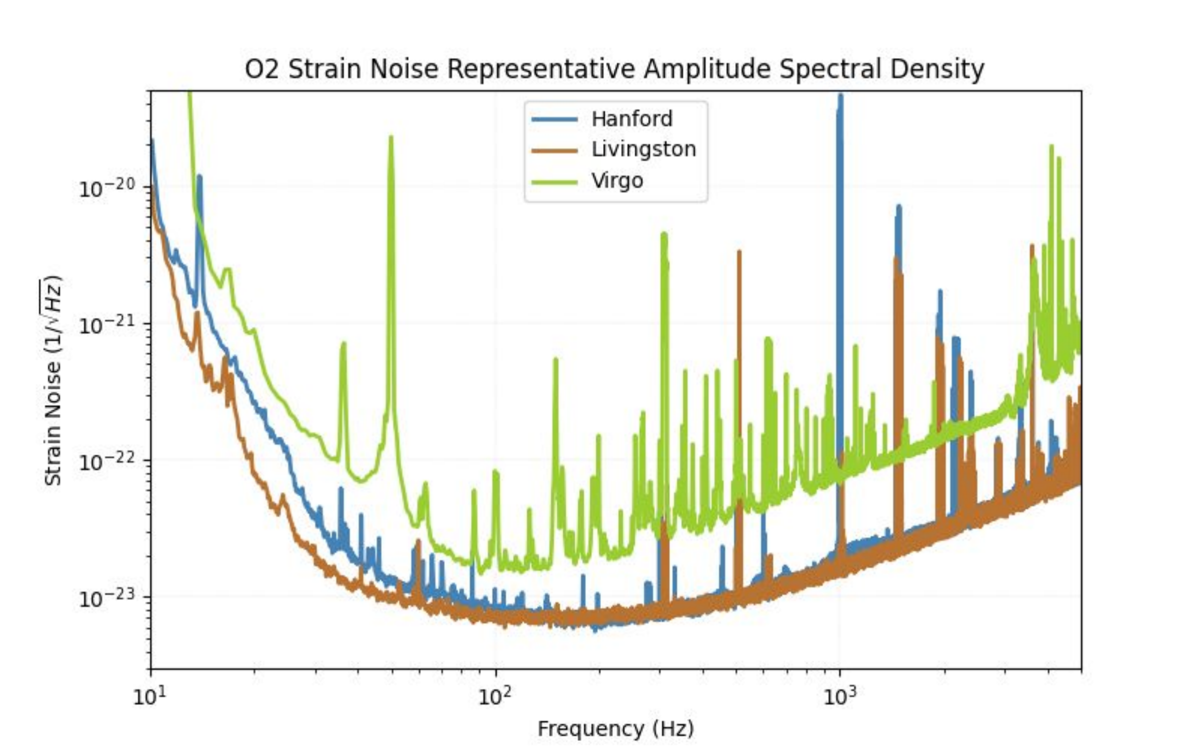
Modelling the Noise

- For a given run, the LVC chooses a **quiet** time series interval during the interferometer's operation
- If no signals have been found at sufficiently high confidence, the interval's amplitude spectral density works as a good enough model of the overall noise



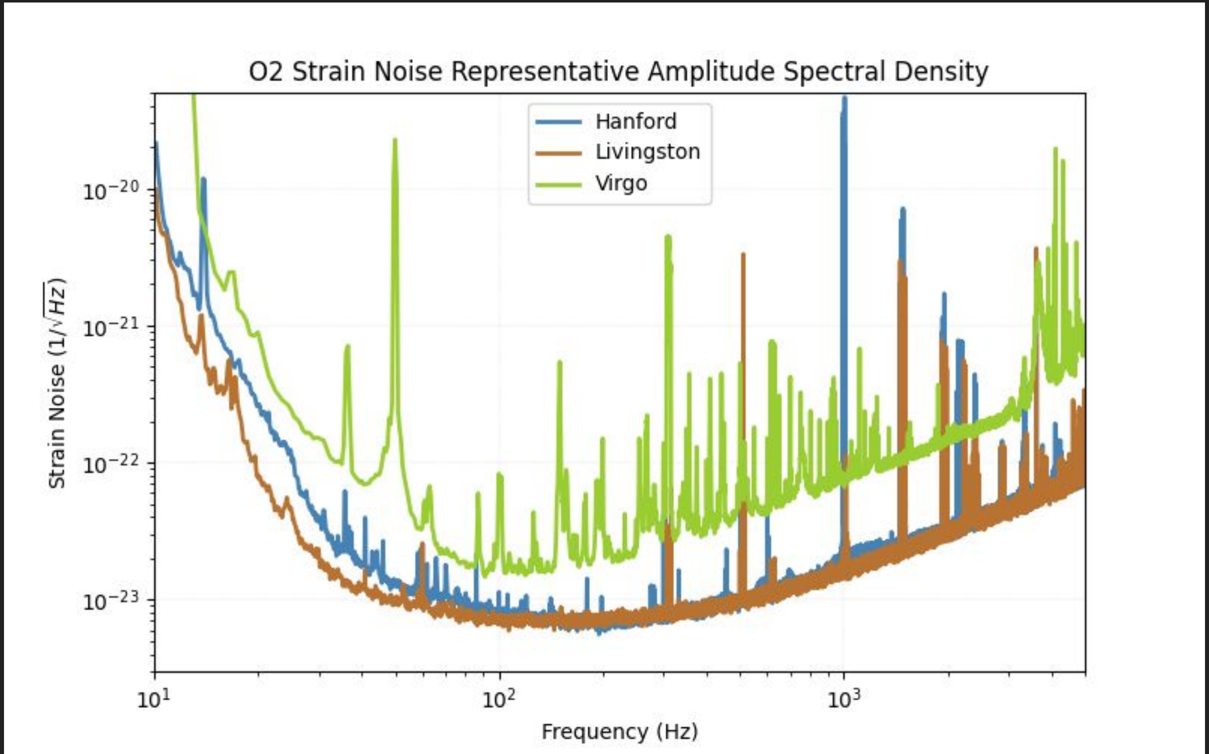
Modelling the Noise

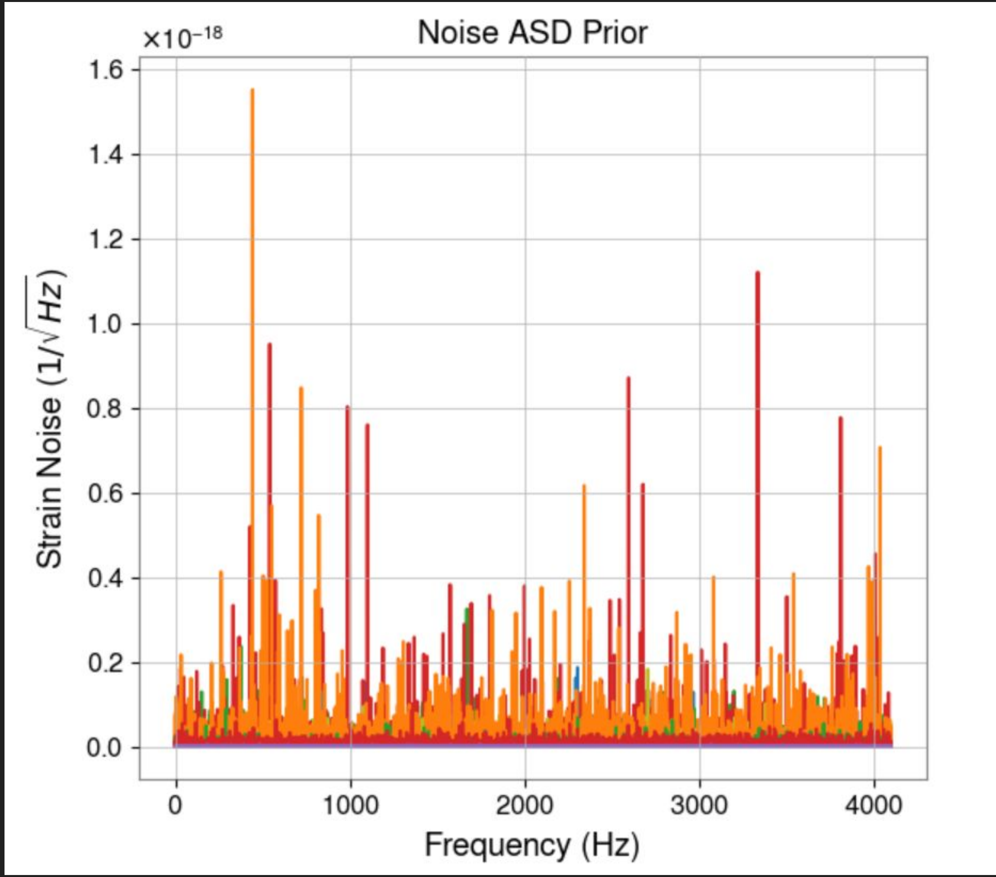
- For a given run, the LVC chooses a **quiet** time series interval during the interferometer's operation
- If no signals have been found at sufficiently high confidence, the interval's amplitude spectral density works as a good enough model of the overall noise
- This works because the strain data has a very low signal-to-noise ratio



Modelling the Noise

Upshot: We can use another correlated field in our measurement equation to *learn* the noise model!



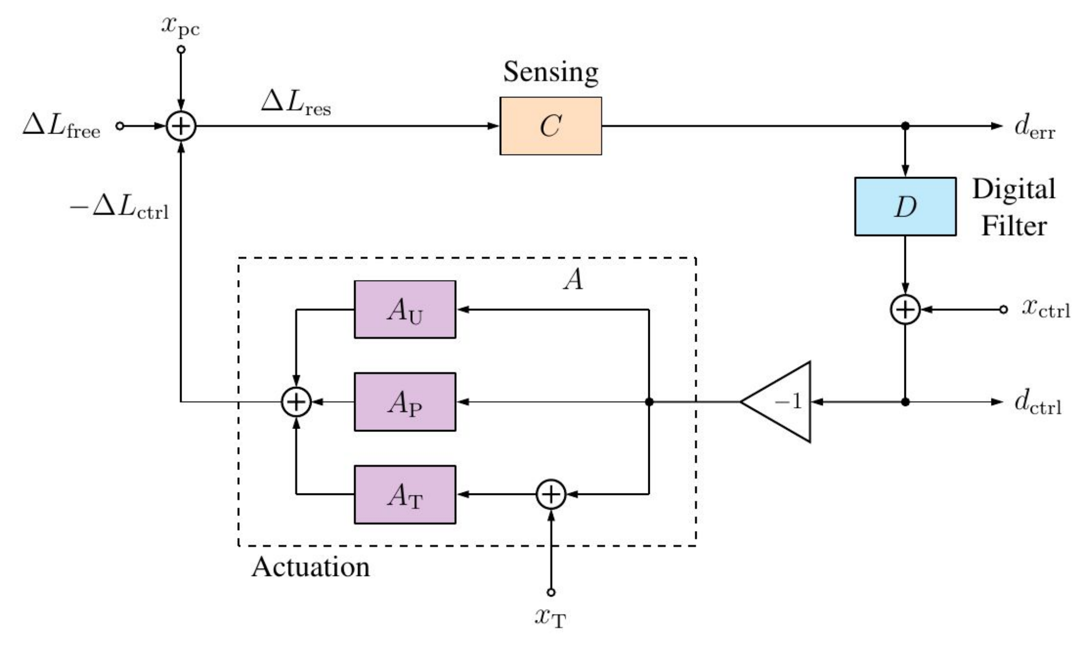


Noise ASD Prior
×10^−18
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
Strain Noise (1/√Hz)
0
1000
2000
3000
4000
Frequency (Hz)

Instrument Response

The Open-loop Control Diagram

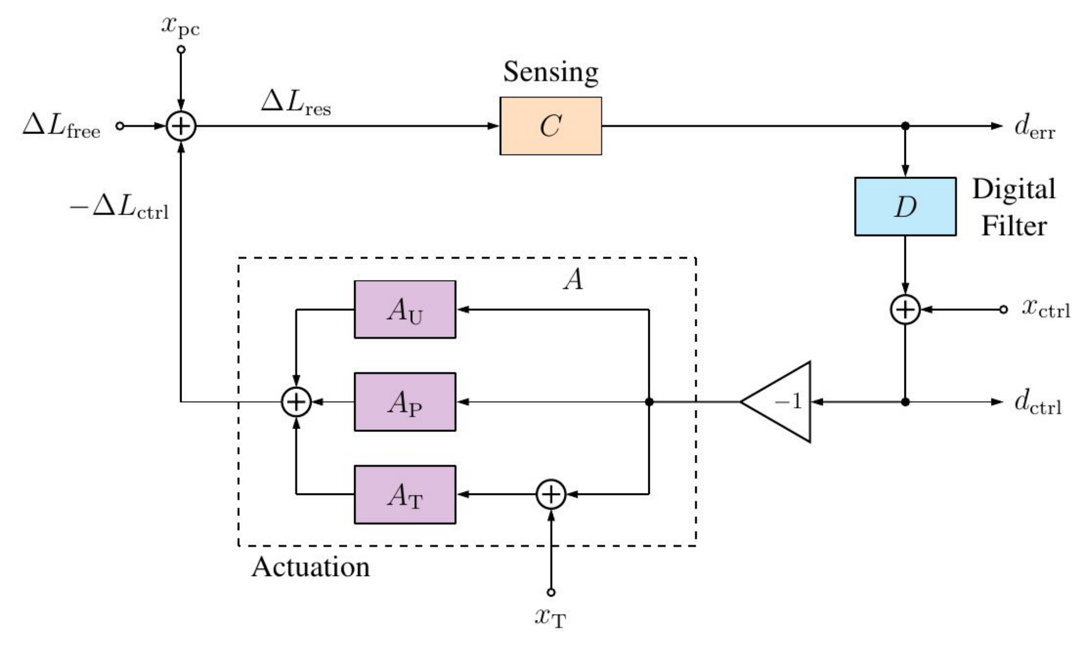
To keep the interferometer in resonance, actuation on the multi-stage pendula suspending the test masses is necessary:



The Open-loop Control Diagram

To keep the interferometer in resonance, actuation on the multi-stage pendula suspending the test masses is necessary:

1. Subtracting a controlled differential arm length ΔL_{ctrl} from the free-running changes in the differential arm length, ΔL_{free}, produces a suppressed signal ΔL_{res}



The Open-loop Control Diagram

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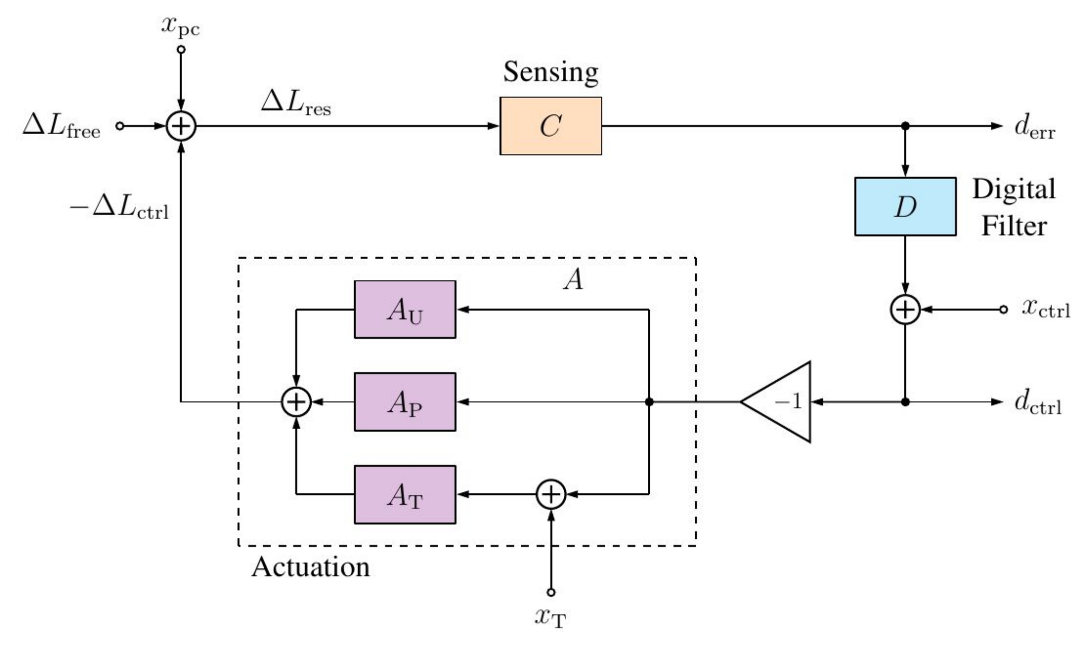
1. Subtracting a controlled differential arm length ΔL_{ctrl} from the free-running changes in the differential arm length, ΔL_{free}, produces a suppressed signal ΔL_{res}
2. Applying a sensing function to the residual displacement, ΔL_{res}, produces the digital error signal, d_{err}, containing both astrophysical signals and displacement noise



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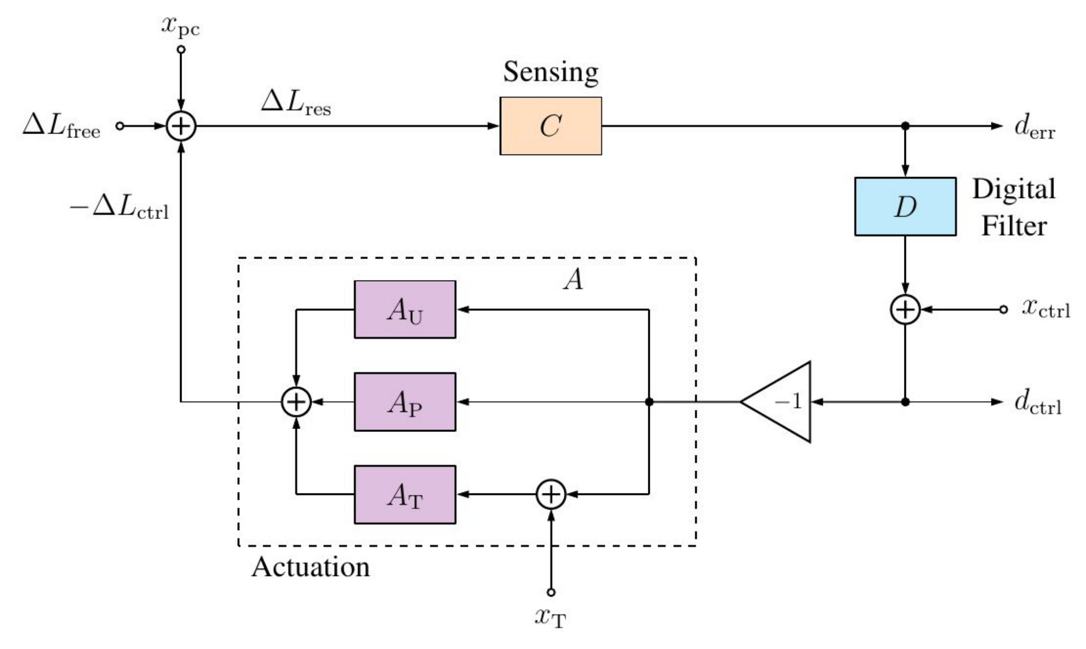
1. Subtracting a controlled differential arm length ΔL_{ctrl} from the free-running changes in the differential arm length, ΔL_{free}, produces a suppressed signal ΔL_{res}
2. Applying a sensing function to the residual displacement, ΔL_{res}, produces the digital error signal, d_{err}, containing both astrophysical signals and displacement noise
3. Applying digital filters to the d_{err} signal produces the control signal d_{ctrl} which governs the actuators' behavior

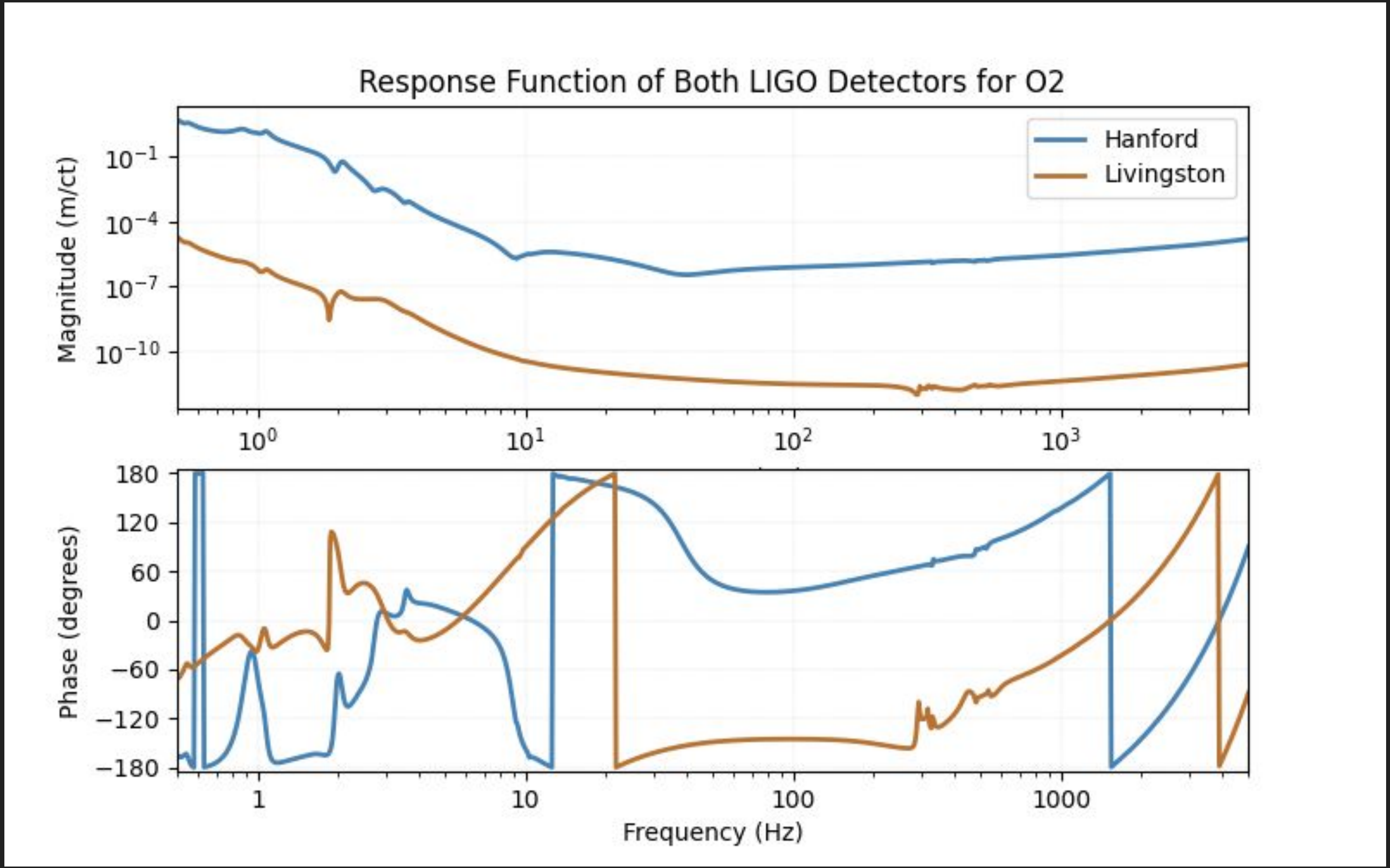


The Response Function

$$\Delta L_{free} = R d_{err}$$

The Response Function

$$\Delta L_{free} = \frac{1 + ADC}{C} d_{err}$$



Response Function of Both LIGO Detectors for O2
Hanford
Livingston
Magnitude (m/ct)
10^{-1}
10^{-4}
10^{-7}
10^{-10}
10^{0}
10^{1}
10^{2}
10^{3}
Phase (degrees)
180
120
60
0
−60
−120
−180
1
10
100
1000
Frequency (Hz)

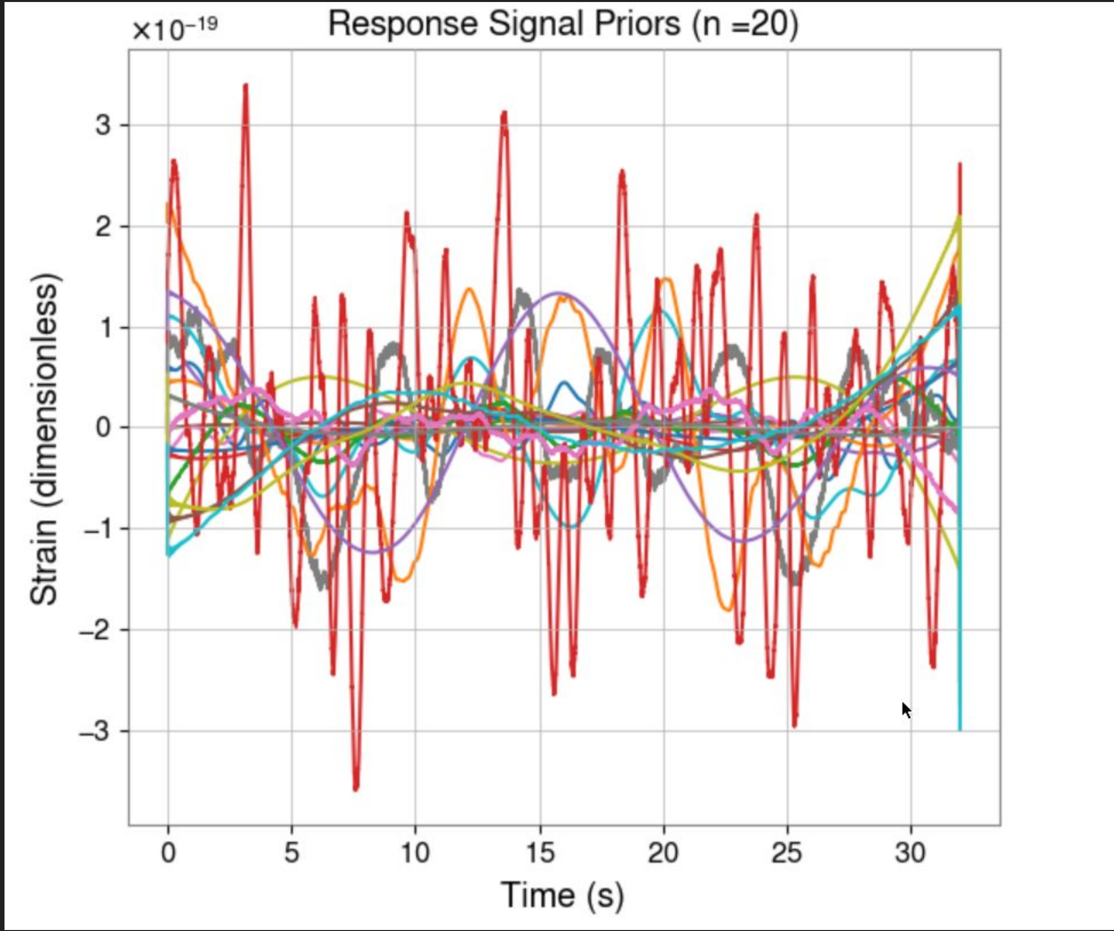
Our Measurement Equation

$$d(f) = (1 + ADC)s_0 + n'$$

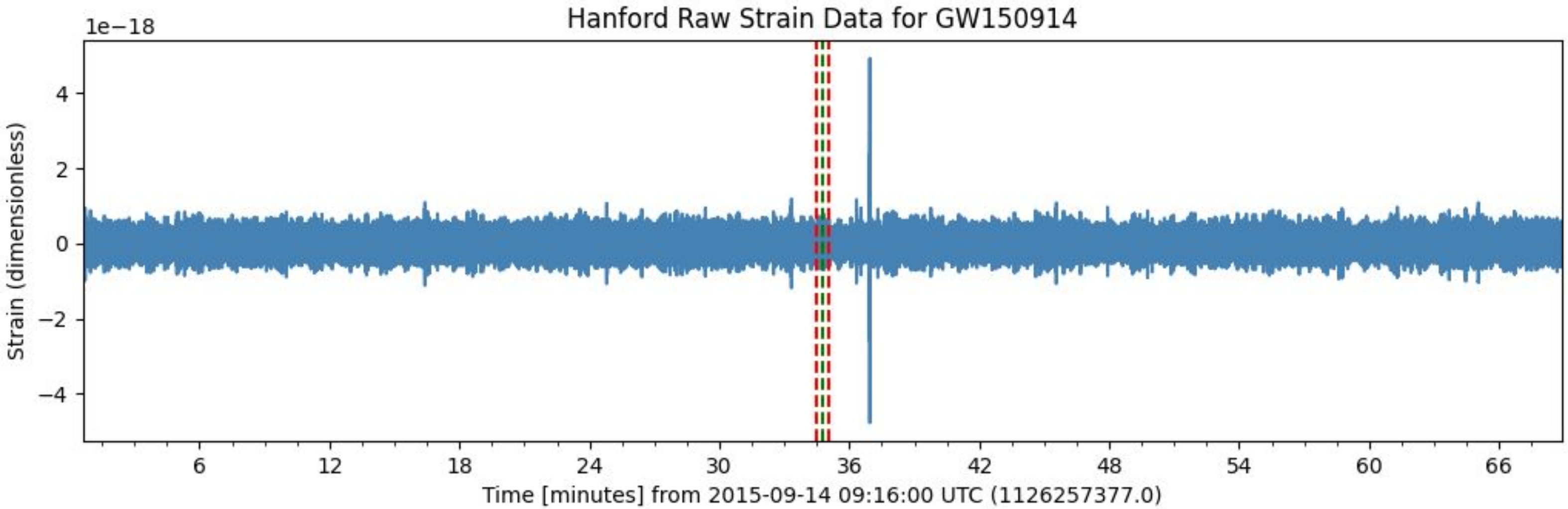
Results

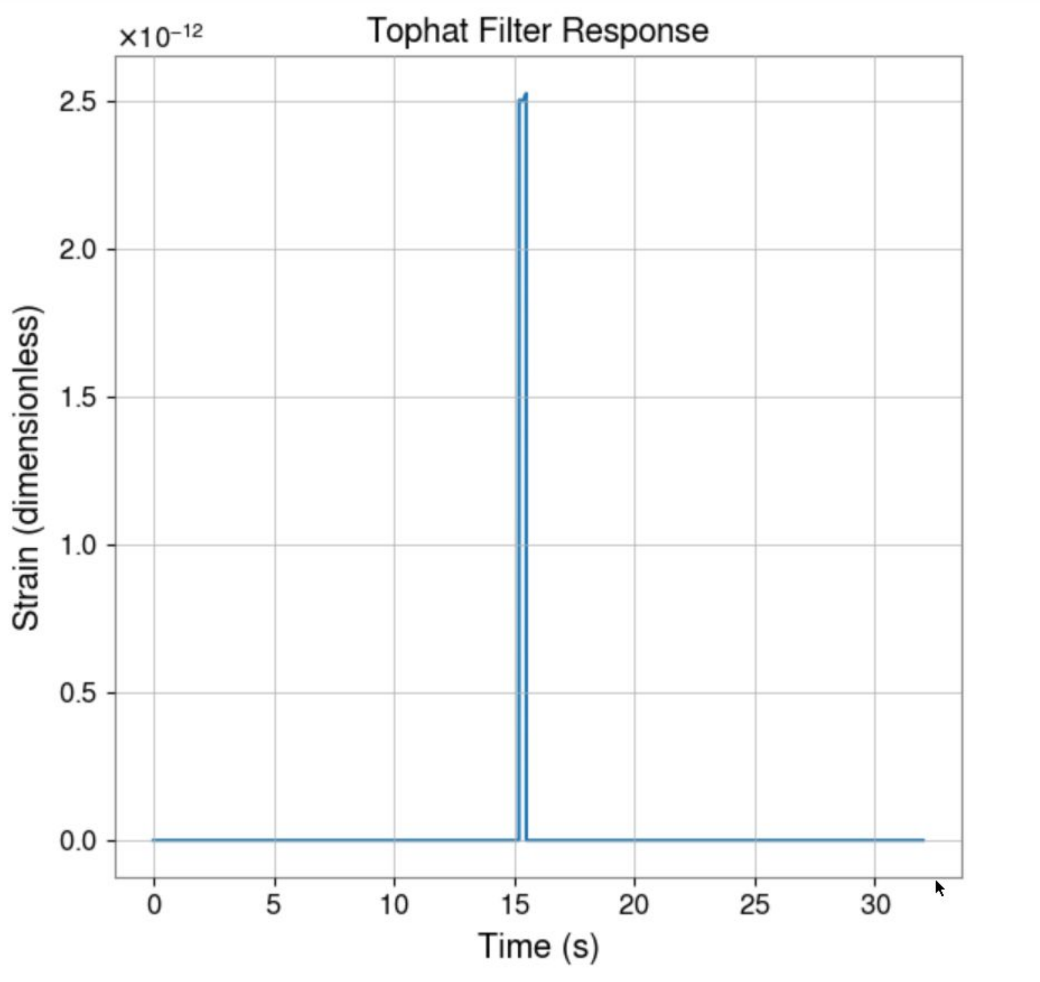
(Preliminary) Results


UNDER
CONSTRUCTION

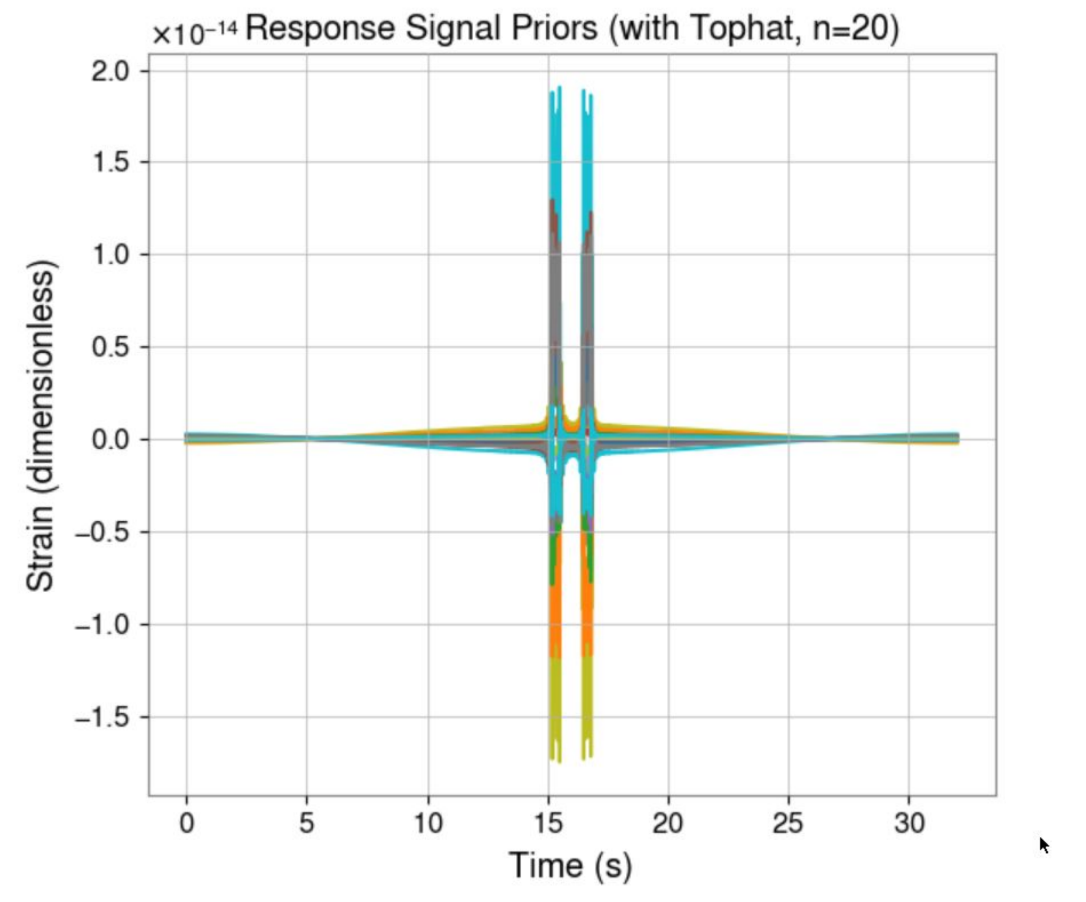


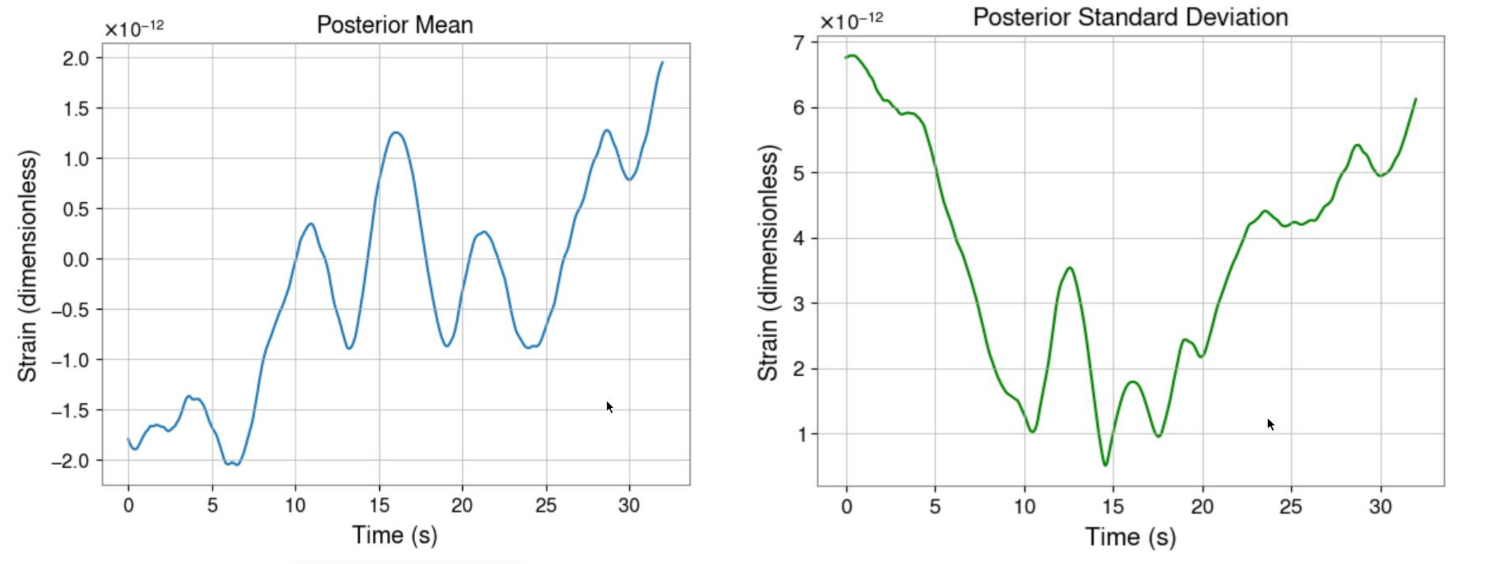
Response Signal Priors (n =20)
×10^−19
Strain (dimensionless)
3
2
1
0
−1
−2
−3
0
5
10
15
20
25
30
Time (s)


Hanford Raw Strain Data for GW150914
1e−18
4
2
0
−2
−4
Strain (dimensionless)
6
12
18
24
30
36
42
48
54
60
66
Time [minutes] from 2015-09-14 09:16:00 UTC (1126257377.0)



Tophat Filter Response
×10⁻¹²
Strain (dimensionless)
2.5
2.0
1.5
1.0
0.5
0.0
0
5
10
15
20
25
30
Time (s)


×10⁻¹⁴ Response Signal Priors (with Tophat, n=20)
Strain (dimensionless)
Time (s)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
5
10
15
20
25
30



Posterior Mean
×10⁻¹²
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Strain (dimensionless)
0
5
10
15
20
25
30
Time (s)
Posterior Standard Deviation
×10⁻¹²
7
6
5
4
3
2
1
Strain (dimensionless)
0
5
10
15
20
25
30
Time (s)

The problem

Can we find a gravitational wave signal in the calibrated strain data *without* relying on any numerical relativity templates?

The problem

Most likely, but we have to improve our noise covariance and instrument response models first!

EXPLORER ···

GRAVITATIONAL_WAVES
- __pycache__
- .ipynb_checkpoints
- cache
- data_sets
- exploratory_notebooks
- ms_thesis
- plots
- reference_scripts
- tests
- .gitignore
- forward_model.py
- get_imprs_plots.ipynb
- gwosc_gestalt.py
- instrument_noise.py
- instrument_responses.py
- inverse_problem.py
- tophat_run_testing.ipynb

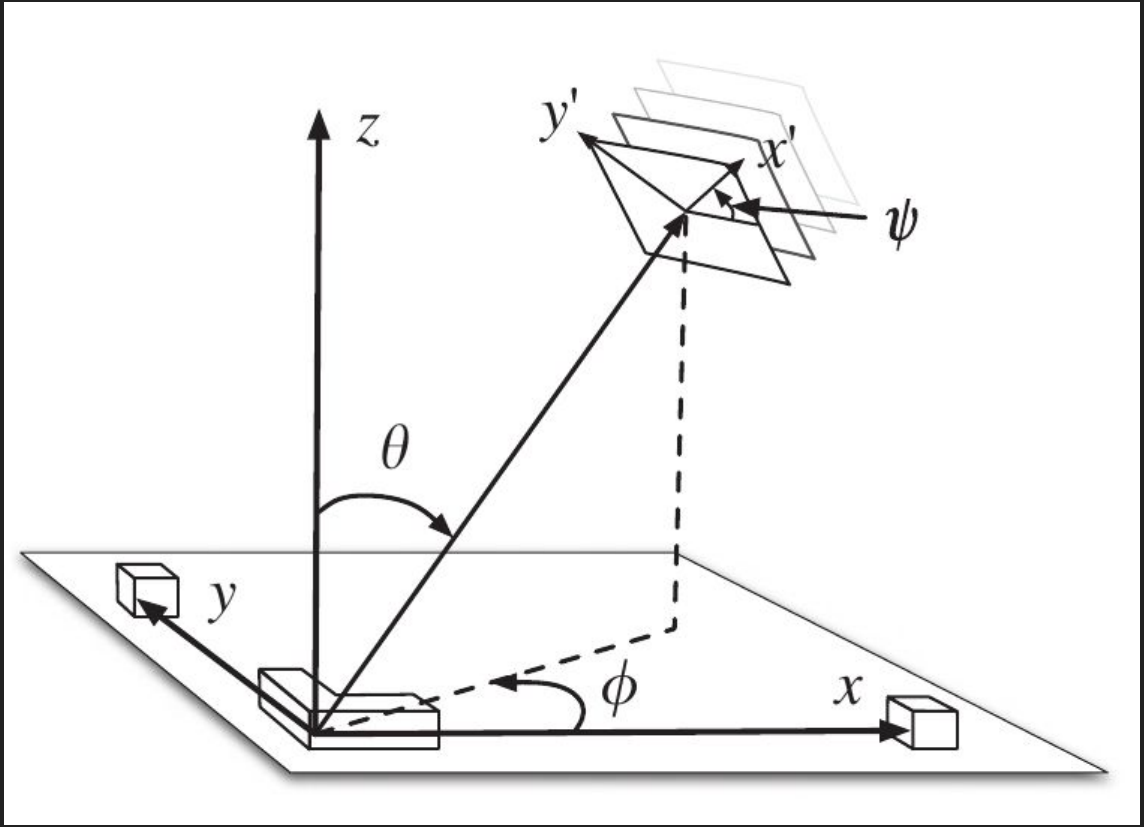
forward_model.py × | instrument_responses.py | instrument_noise.py

forward_model.py

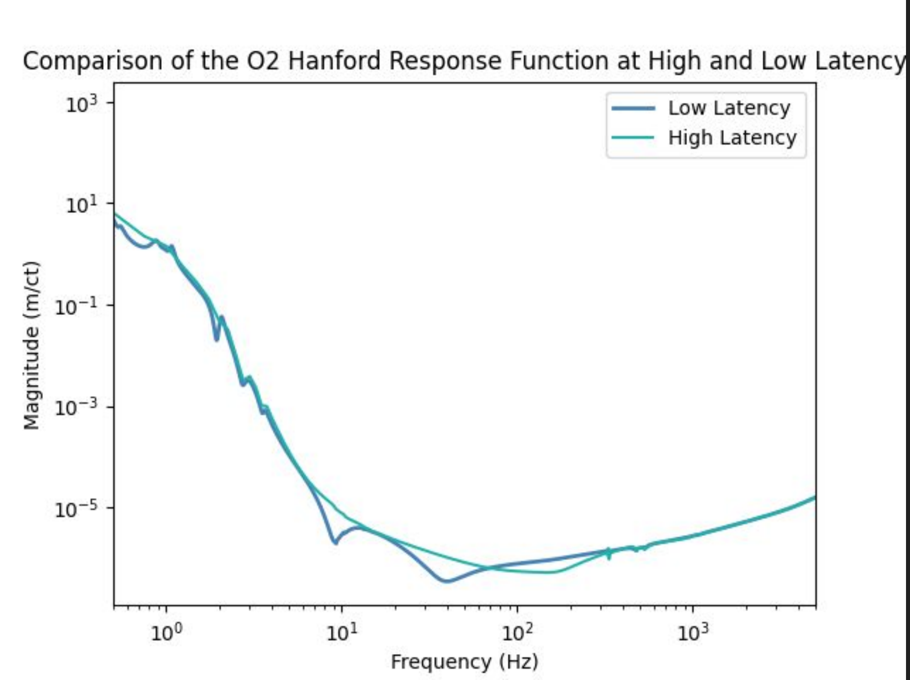
```python
ic_newton = ift.AbsDeltaEnergyController(name='Newton', deltaE=0.5,
                                         convergence_level=2, iteration_limit=
ic_sampling_nl = ift.AbsDeltaEnergyController(name='Sampling (nonlin)',
                                              deltaE=0.5, iteration_limit=15,
                                              convergence_level=2)
minimizer = ift.NewtonCG(ic_newton)
minimizer_sampling = ift.NewtonCG(ic_sampling_nl)

# We use variable covariance Gaussian energy because we want to learn the noi

# We first need to define the keys
noise_operator = field_u_operator.power(-1).ducktape_left('inverse_covariance


# we are getting r = Rs - d and ducktape
residuals = (ift.Adder(FFT_operator(strain_field), neg=True) @ instrument_resp
residual_noise = noise_operator + residuals

# define the likelihoood energy and apply it to r
likelihood_energy = ift.VariableCovarianceGaussianEnergy(domain=frequency_dom
                    inverse_covariance_key='inverse_covariance',
                    sampling_dtype=np.float64, use_full_fisher=True) @ residu

# Minimize the Kullback-Leibler divergence
n_iterations = 1
n_samples = lambda iiter: 10 if iiter < 5 else 20
samples = ift.optimize_kl(likelihood_energy, n_iterations, n_samples,
                          minimizer, ic_sampling, minimizer_sampling,
                          output_directory="plots/tophat_filter", # TO DO: ad
                          comm=None)
```

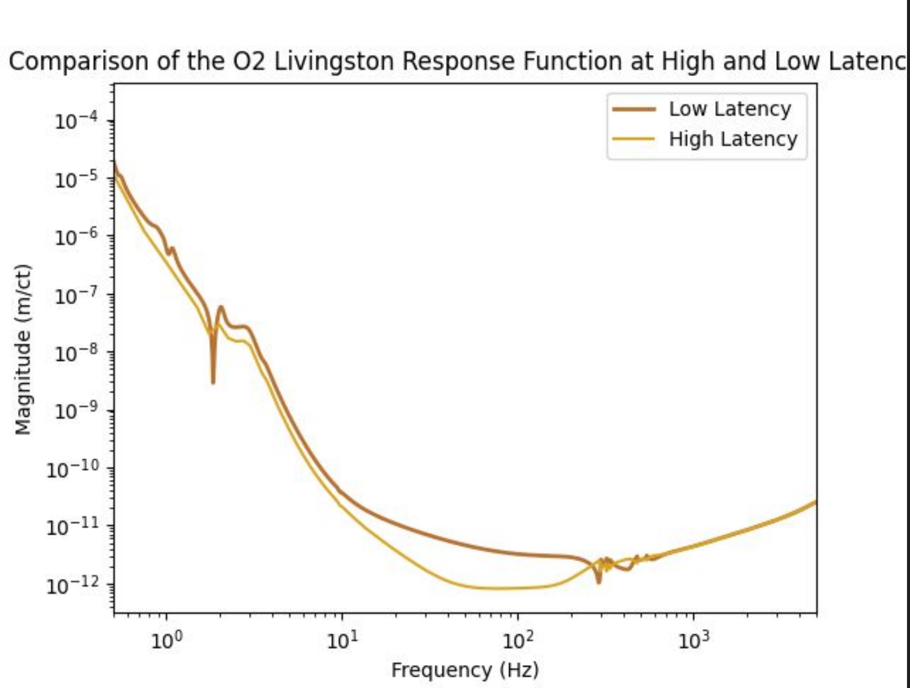
Thanks!

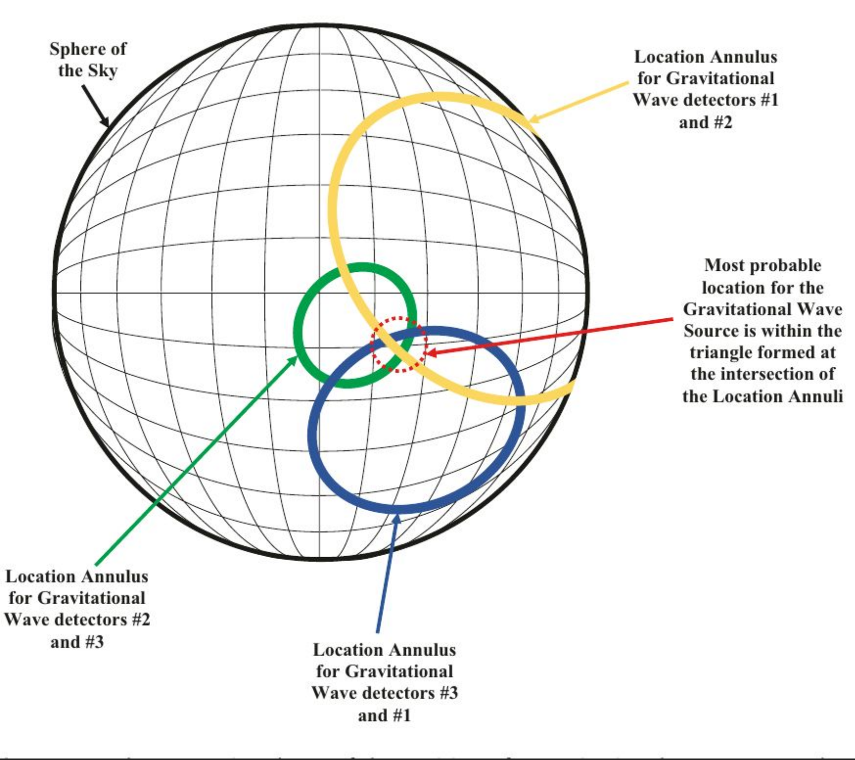
Backup Slides

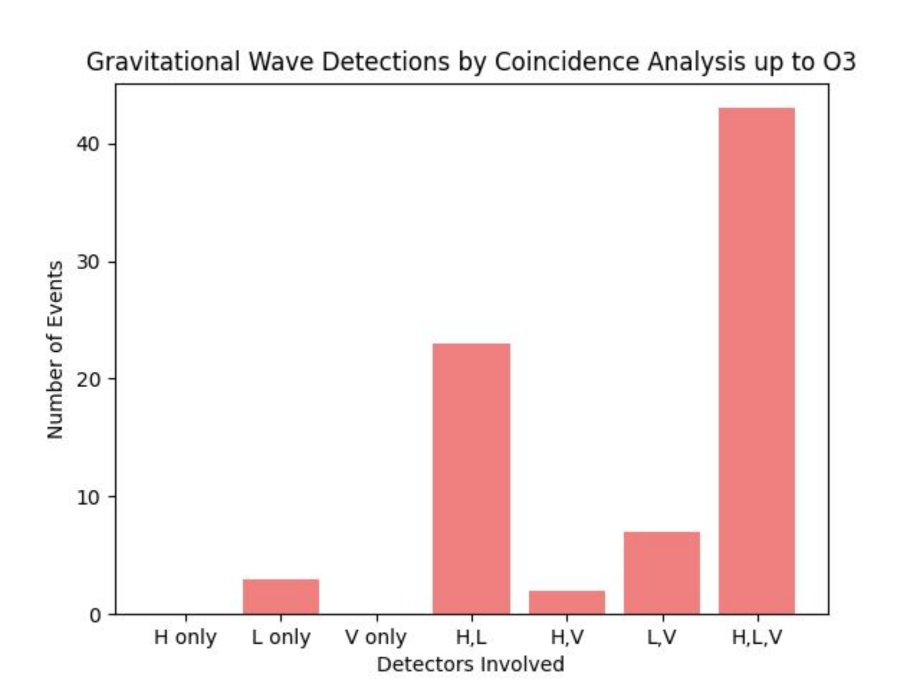


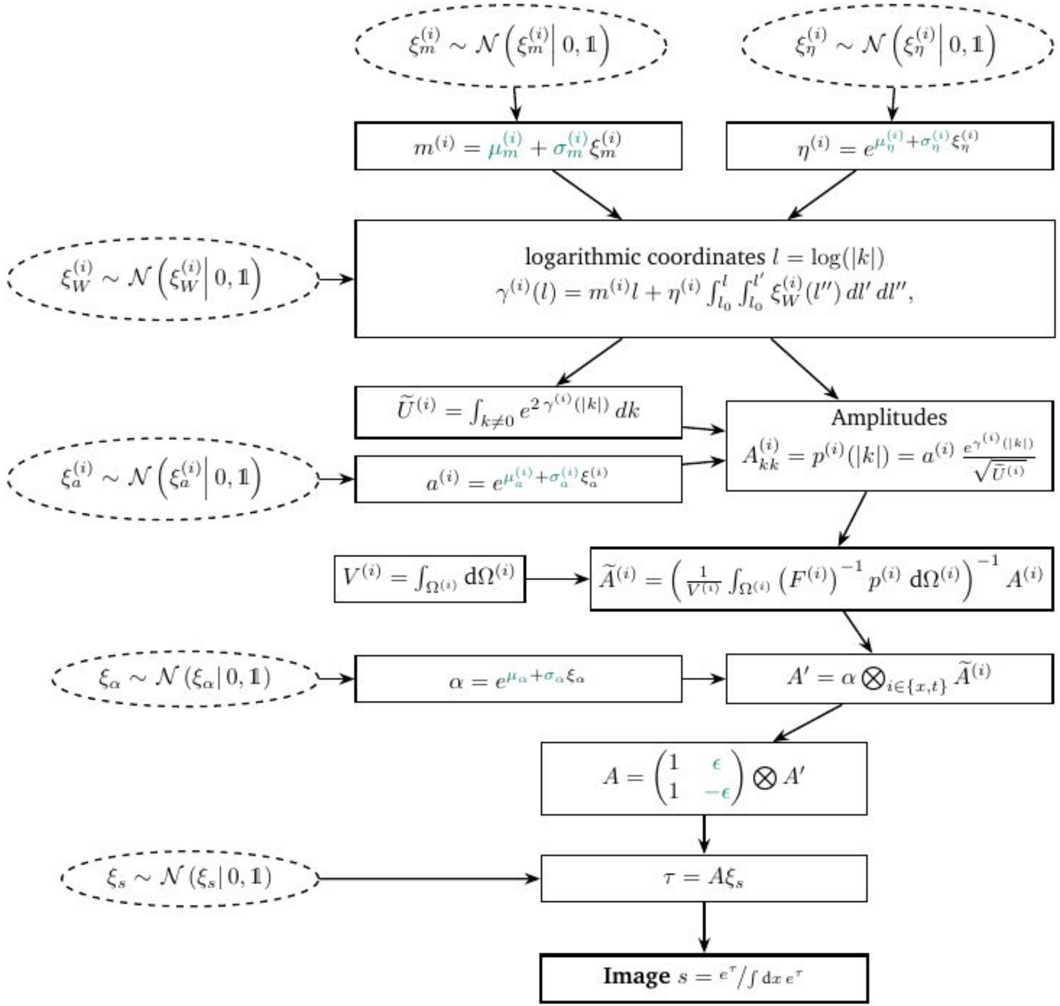
z
y'
x'
ψ
θ
y
ϕ
x


Comparison of the O2 Hanford Response Function at High and Low Latency
Low Latency
High Latency
Magnitude (m/ct)
Frequency (Hz)
10^{3}
10^{1}
10^{-1}
10^{-3}
10^{-5}
10^{0}
10^{1}
10^{2}
10^{3}


Comparison of the O2 Livingston Response Function at High and Low Latenc
Low Latency
High Latency
Magnitude (m/ct)
Frequency (Hz)
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{0}
10^{1}
10^{2}
10^{3}


Sphere of the Sky
Location Annulus for Gravitational Wave detectors #1 and #2
Most probable location for the Gravitational Wave Source is within the triangle formed at the intersection of the Location Annuli
Location Annulus for Gravitational Wave detectors #2 and #3
Location Annulus for Gravitational Wave detectors #3 and #1


Gravitational Wave Detections by Coincidence Analysis up to O3
Number of Events
0
10
20
30
40
H only
L only
V only
H,L
H,V
L,V
H,L,V
Detectors Involved



$\xi_m^{(i)} \sim \mathcal{N}\left(\xi_m^{(i)} \middle| 0, \mathbb{1}\right)$
$\xi_\eta^{(i)} \sim \mathcal{N}\left(\xi_\eta^{(i)} \middle| 0, \mathbb{1}\right)$
$m^{(i)} = \mu_m^{(i)} + \sigma_m^{(i)} \xi_m^{(i)}$
$\eta^{(i)} = e^{\mu_\eta^{(i)} + \sigma_\eta^{(i)} \xi_\eta^{(i)}}$
$\xi_W^{(i)} \sim \mathcal{N}\left(\xi_W^{(i)} \middle| 0, \mathbb{1}\right)$
logarithmic coordinates $l = \log(|k|)$
$\gamma^{(i)}(l) = m^{(i)} l + \eta^{(i)} \int_{l_0}^{l} \int_{l_0}^{l'} \xi_W^{(i)}(l'')\, dl'\, dl''$,
$\widetilde{U}^{(i)} = \int_{k \neq 0} e^{2\gamma^{(i)}(|k|)}\, dk$
Amplitudes
$A_{kk}^{(i)} = p^{(i)}(|k|) = a^{(i)} \frac{e^{\gamma^{(i)}(|k|)}}{\sqrt{\widetilde{U}^{(i)}}}$
$\xi_a^{(i)} \sim \mathcal{N}\left(\xi_a^{(i)} \middle| 0, \mathbb{1}\right)$
$a^{(i)} = e^{\mu_a^{(i)} + \sigma_a^{(i)} \xi_a^{(i)}}$
$V^{(i)} = \int_{\Omega^{(i)}} \mathrm{d}\Omega^{(i)}$
$\widetilde{A}^{(i)} = \left(\frac{1}{V^{(i)}} \int_{\Omega^{(i)}} \left(F^{(i)}\right)^{-1} p^{(i)}\, \mathrm{d}\Omega^{(i)} \right)^{-1} A^{(i)}$
$\xi_\alpha \sim \mathcal{N}(\xi_\alpha | 0, \mathbb{1})$
$\alpha = e^{\mu_\alpha + \sigma_\alpha \xi_\alpha}$
$A' = \alpha \bigotimes_{i \in \{x,t\}} \widetilde{A}^{(i)}$
$A = \begin{pmatrix} 1 & \epsilon \\ 1 & -\epsilon \end{pmatrix} \otimes A'$
$\xi_s \sim \mathcal{N}(\xi_s | 0, \mathbb{1})$
$\tau = A \xi_s$
Image $s = e^{\tau} / \int \mathrm{d}x\, e^{\tau}$