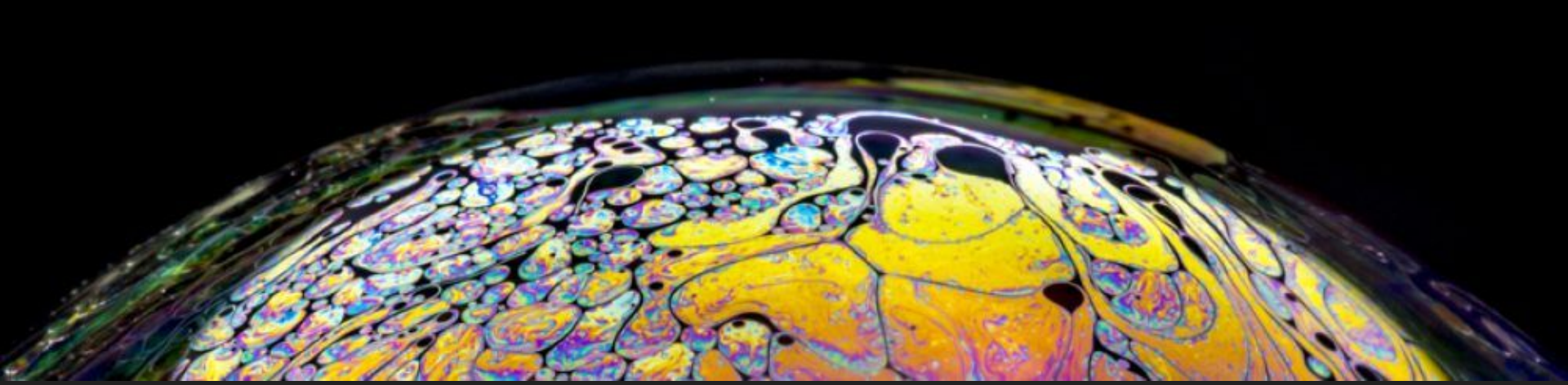


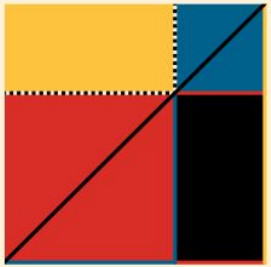
What is the Geometry of Nature?

Sebastián Gil Rodríguez
MCMP

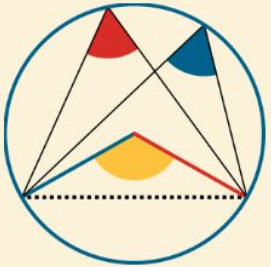
ic.SoAP.2021/EENPS

BOOKS.

BOOK I.
Basic plane geometry

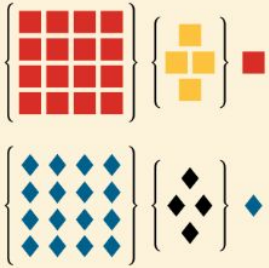


BOOK II.
Geometric algebra



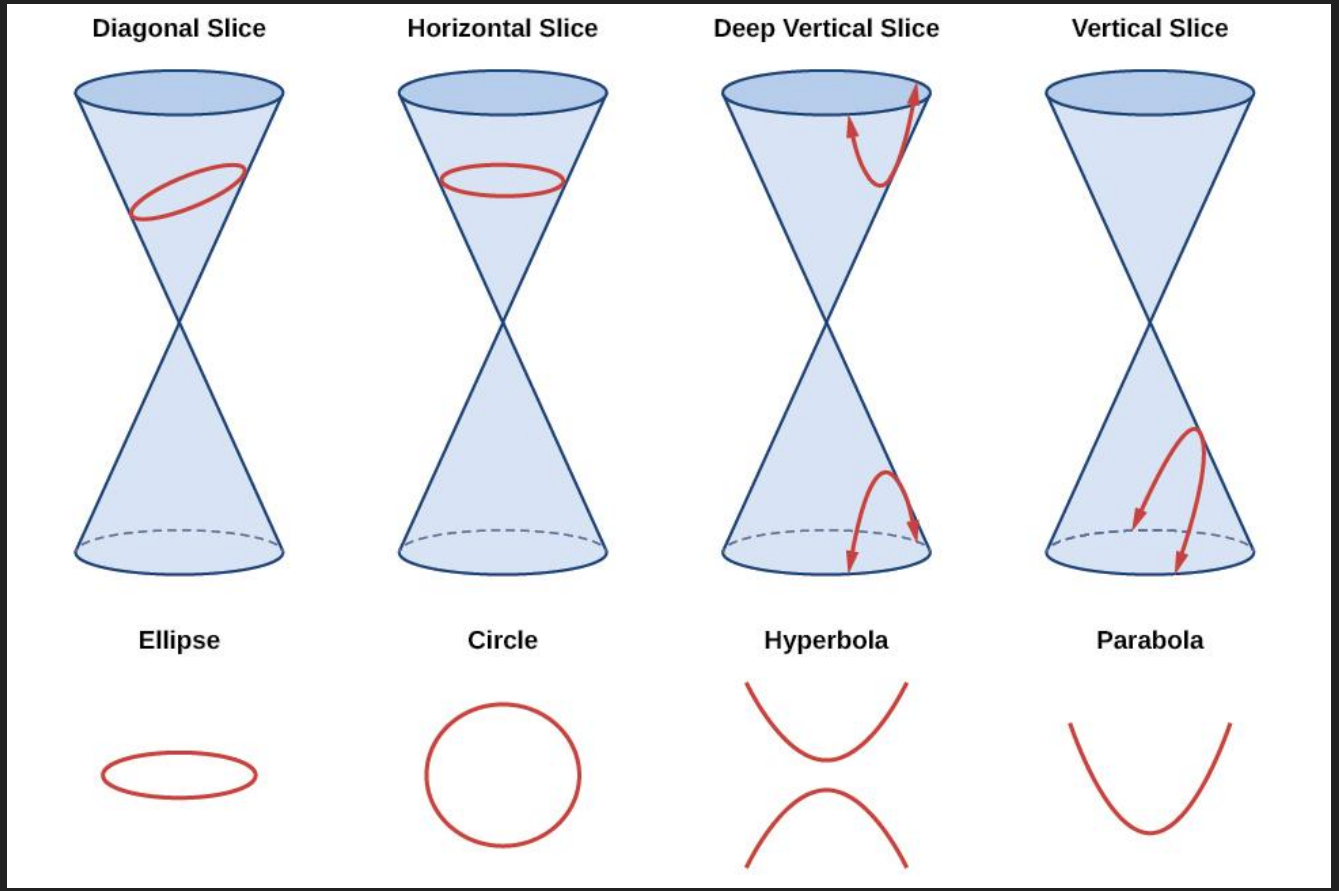
BOOK III.
Circles and angles

BOOK IV.
Regular polygons

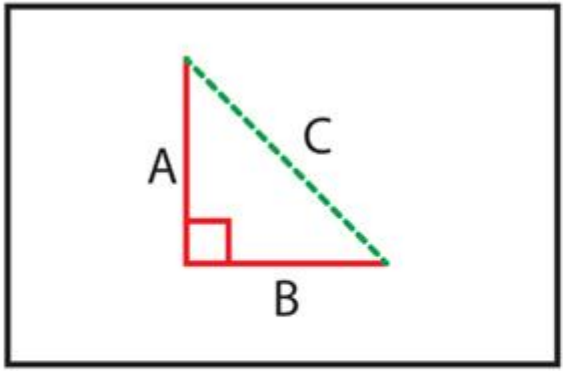


BOOK V.
Ratios and proportions

BOOK VI.
Geometric proportions

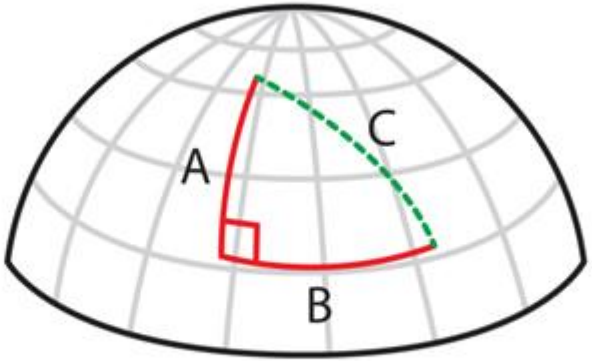

Diagonal Slice
Horizontal Slice
Deep Vertical Slice
Vertical Slice
Ellipse
Circle
Hyperbola
Parabola

“Geometry is a science that flourishes without the help of experience, that is, geometry is a priori. Mathematics presents the most splendid example of the successful extension of pure reason, without the help of experience"



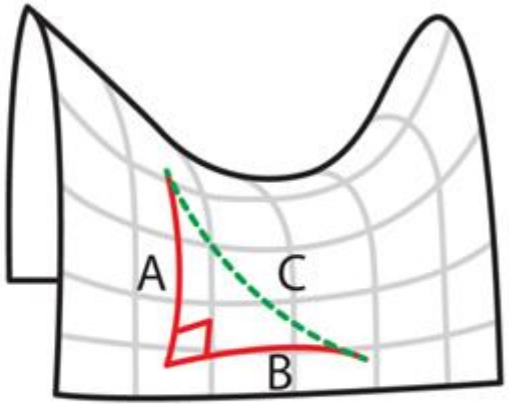
Euclidean

$a^2 + b^2 = c^2$



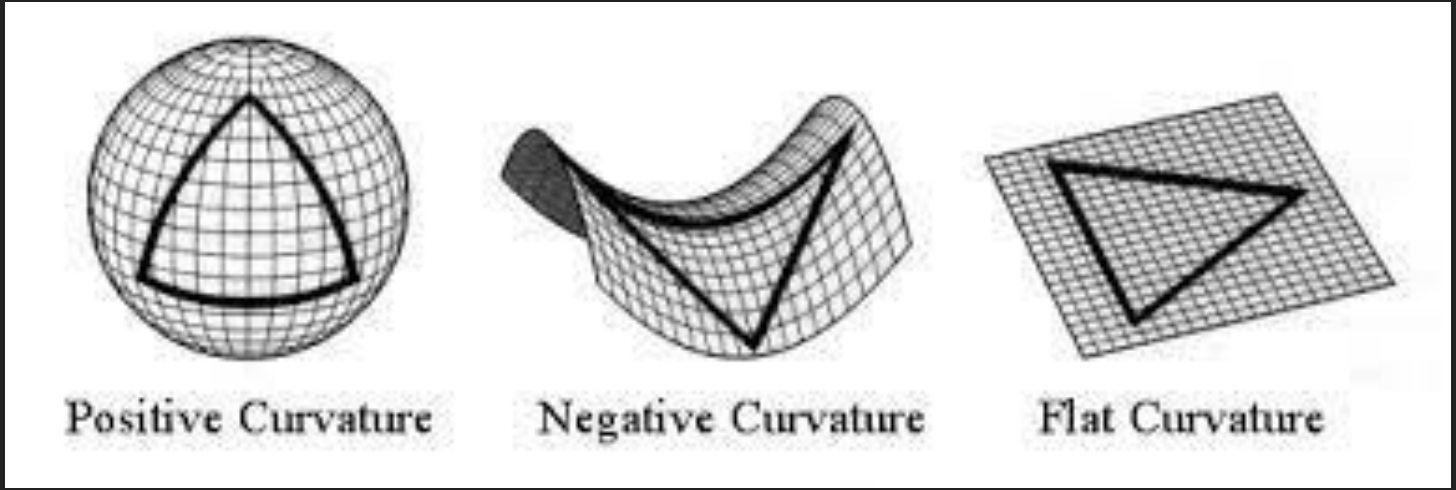
Spherical

$\cos a \times \cos b = \cos c$

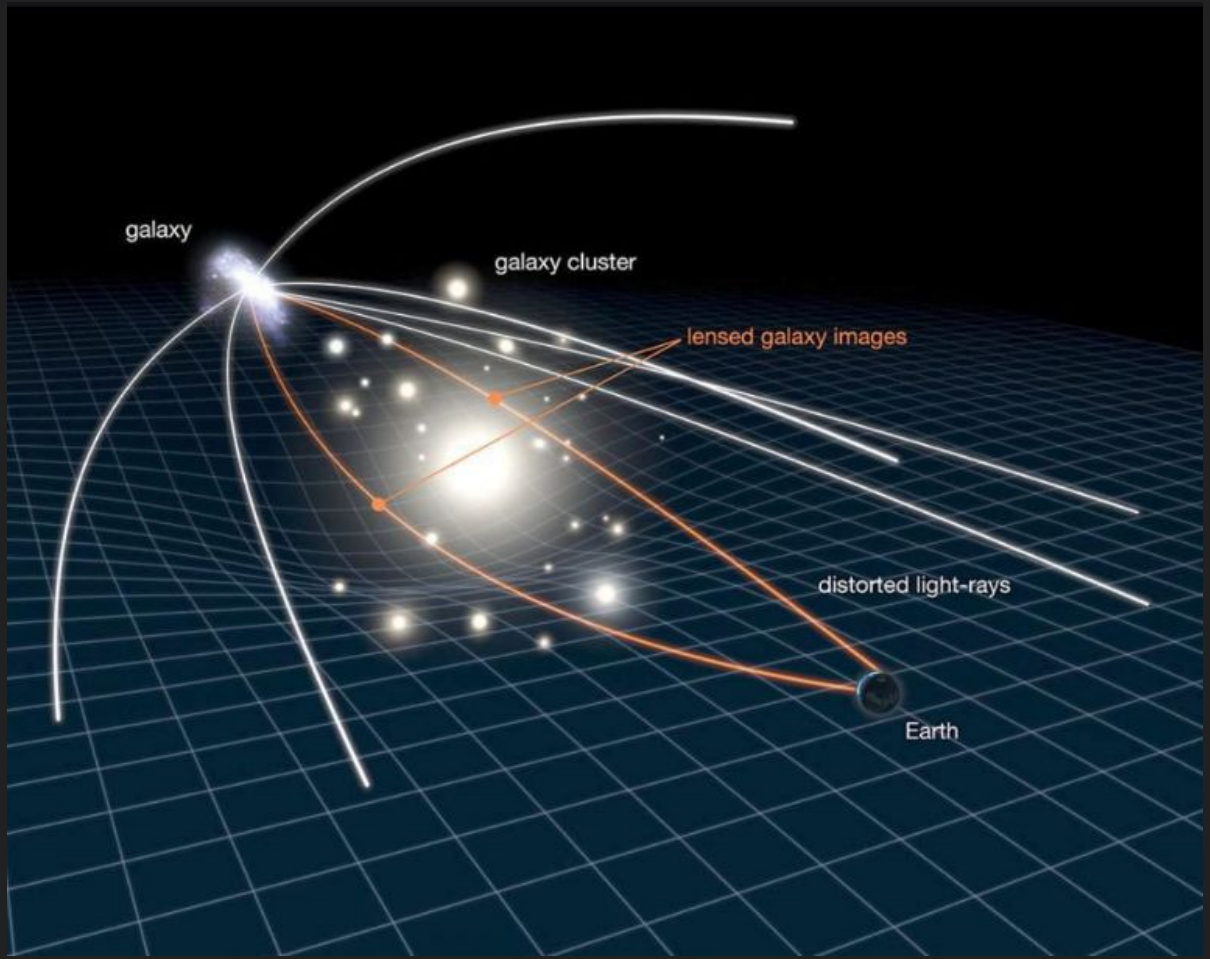


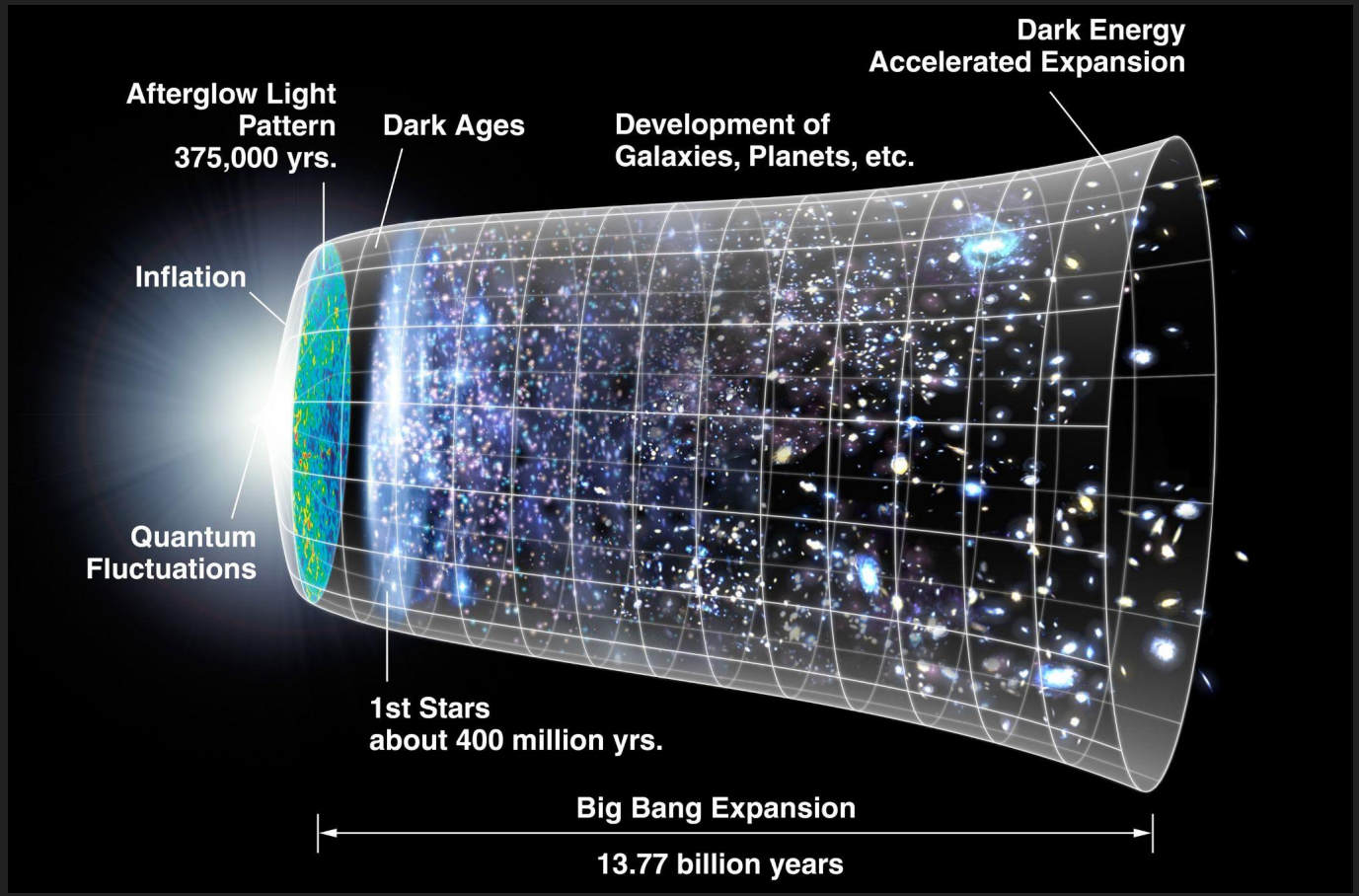
Hyperbolic

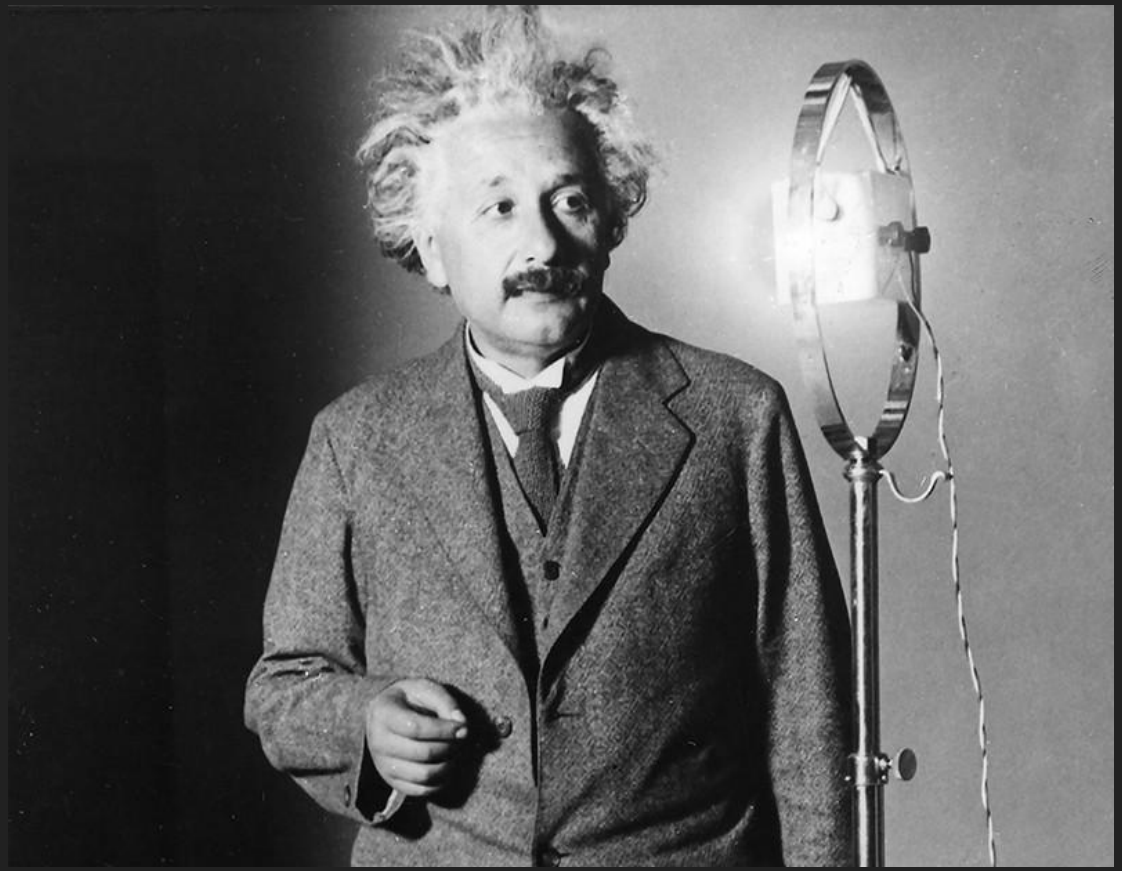
$\cosh a \times \cosh b = \cosh c$


Positive Curvature
Negative Curvature
Flat Curvature

When we want to apply geometry to facts of experience, where it is always only a matter of physical alikeness in value, we can apply only the propositions of that science which I have termed physical geometry. For anyone who derives the axioms from experience, our geometry up to now has indeed been a physical geometry, only one which relies on a great host of experiences collected without any plan, instead of on a system of methodically pursued ones.


galaxy
galaxy cluster
lensed galaxy images
distorted light-rays
Earth


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years



In so far as the propositions of mathematics refer to reality they are not certain; and in so far as they are certain they do not refer to reality. Full clarity about the situation appears to me to have been first obtained in general by that tendency in mathematics known under the name of "axiomatics." The advance achieved by axiomatics consists in having cleanly separated the formal-logical element from the material or intuitive content. According to axiomatics only the formal-logical element constitutes the object of mathematics, but not the intuitive or other content connected with the formal-logical element.”

Uncle Bighat

Lives in a hotel
Always creates problems
Personal space

Sadboi

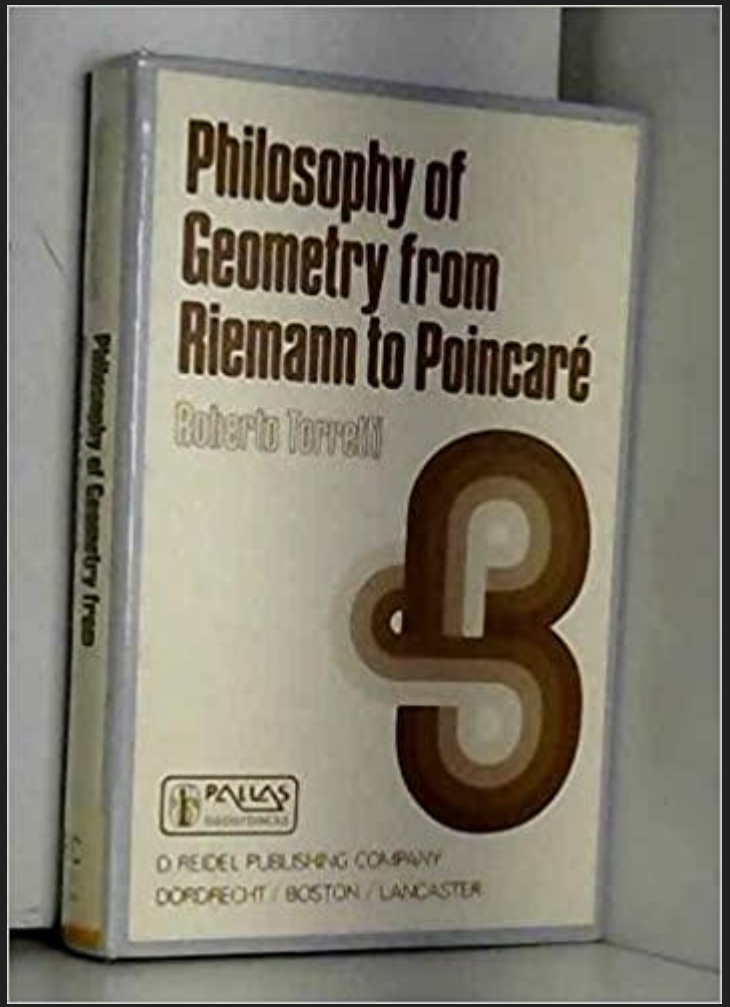
Speaks only in emoji
Lots of regrets
Likes games

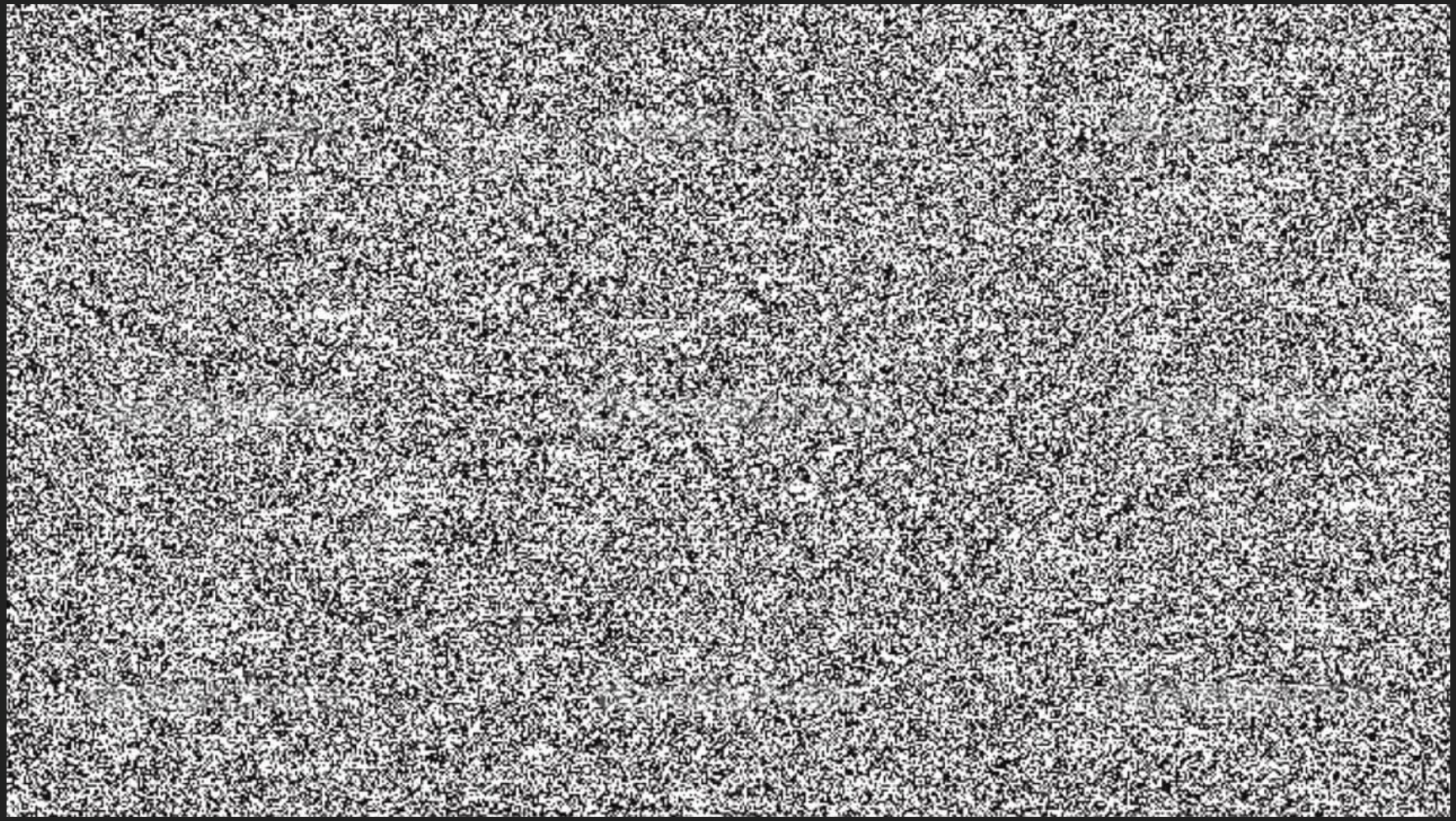
Pipe Dude

Stuck in the past
Has too many books
Not a Harry Potter fan

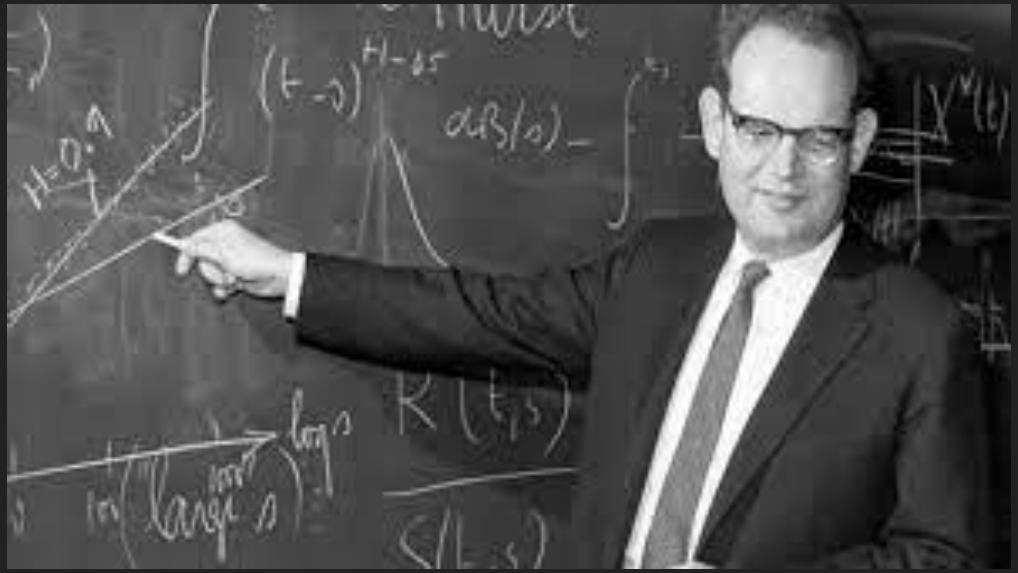
Harry Potter

Good at trivia games
Picks fights with math teacher
Misunderstood

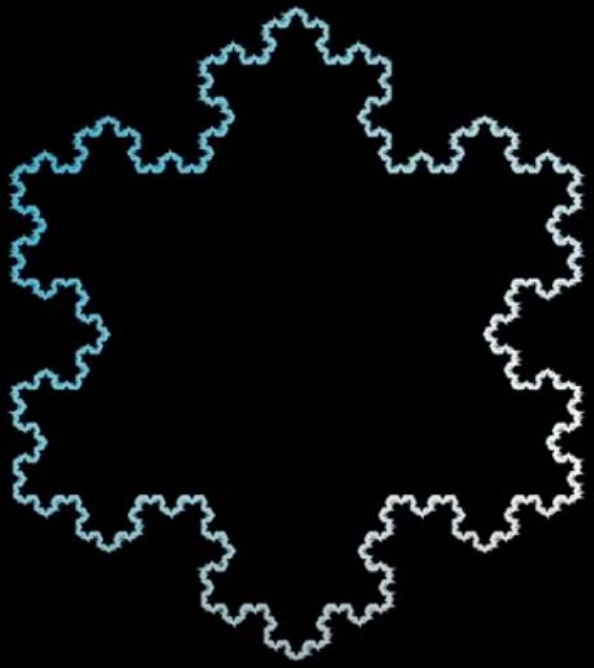

Philosophy of Geometry from Riemann to Poincaré
Roberto Torretti
PALLAS
D. REIDEL PUBLISHING COMPANY
DORDRECHT / BOSTON / LANCASTER



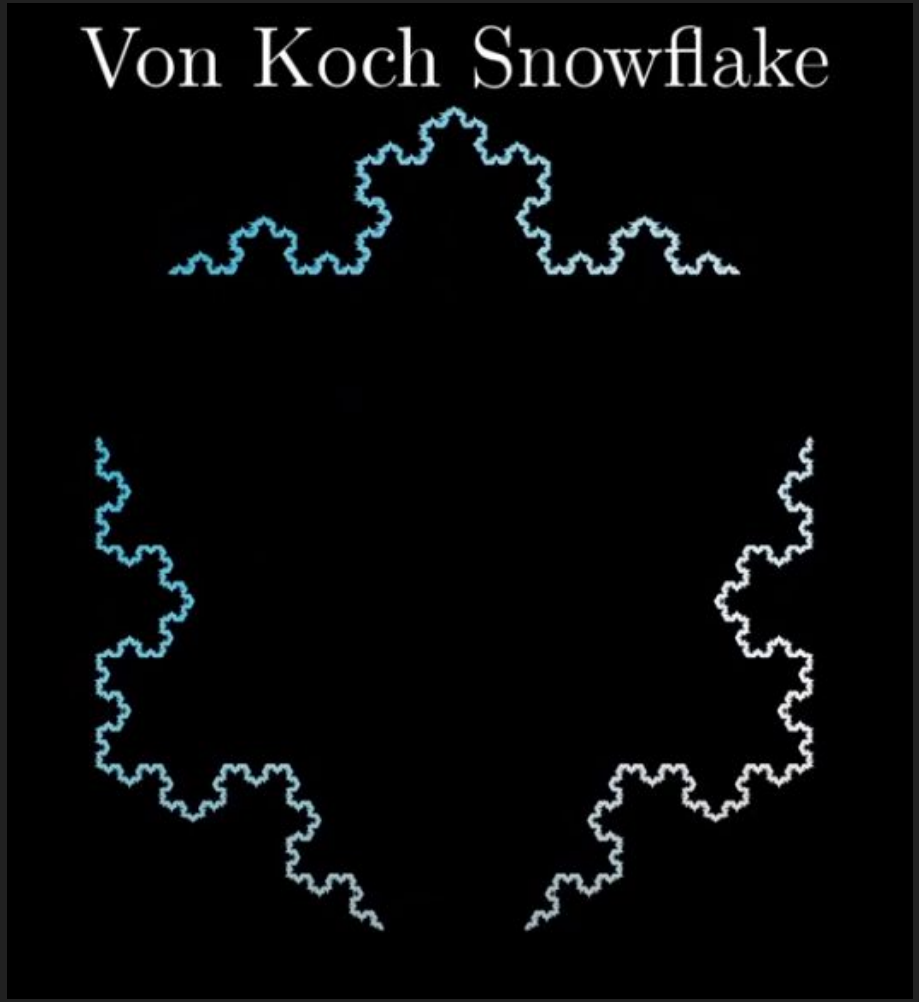
Fractals!



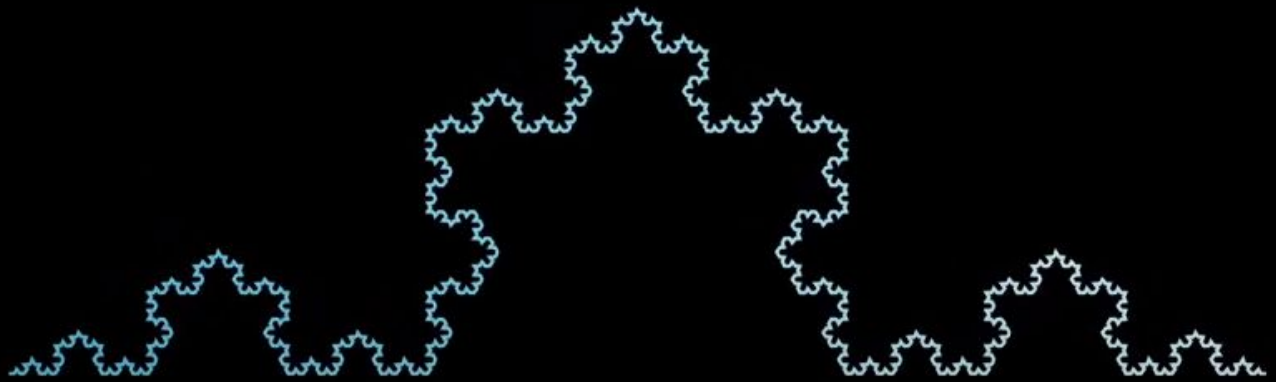
Von Koch Snowflake



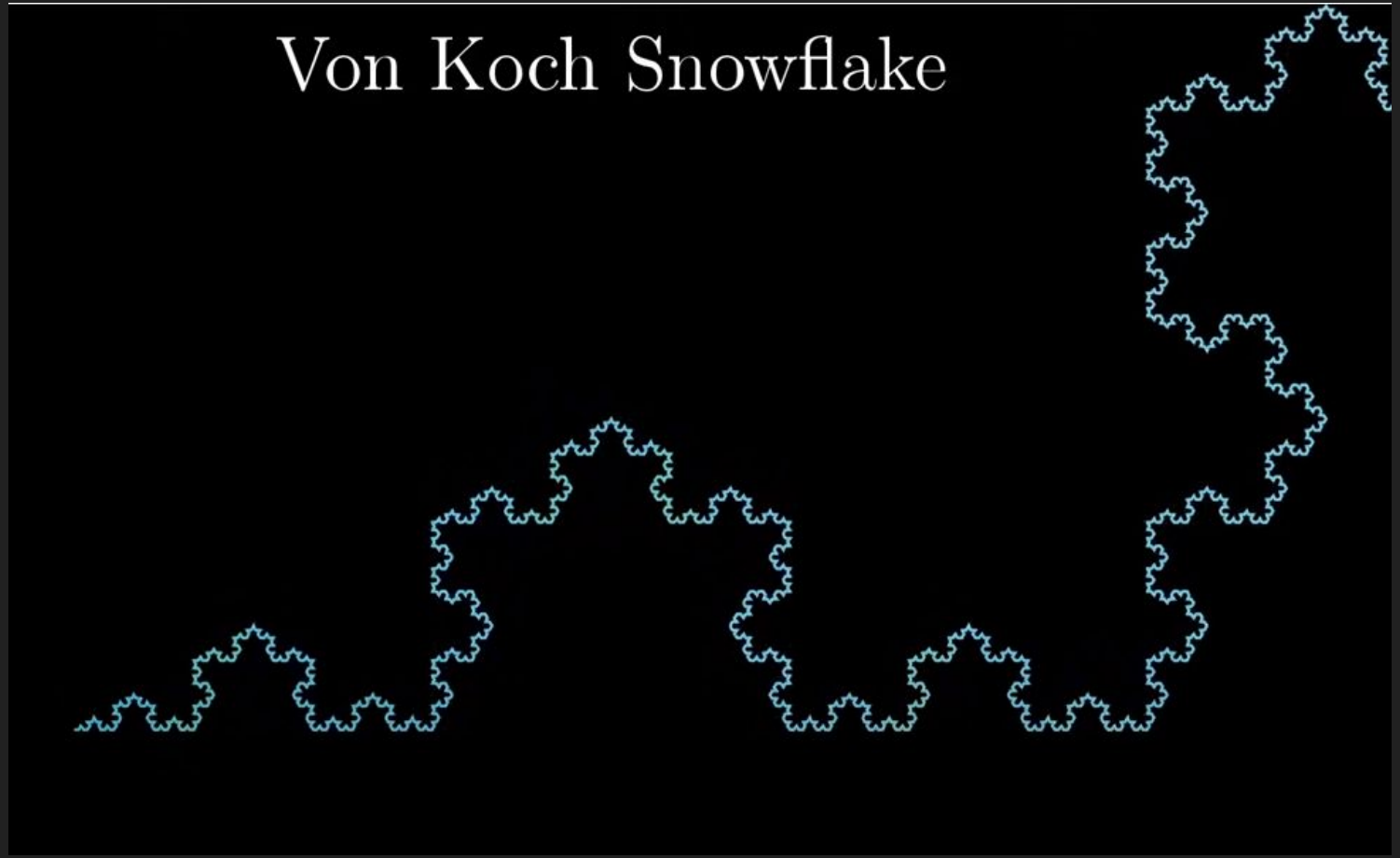
Von Koch Snowflake



Von Koch Snowflake

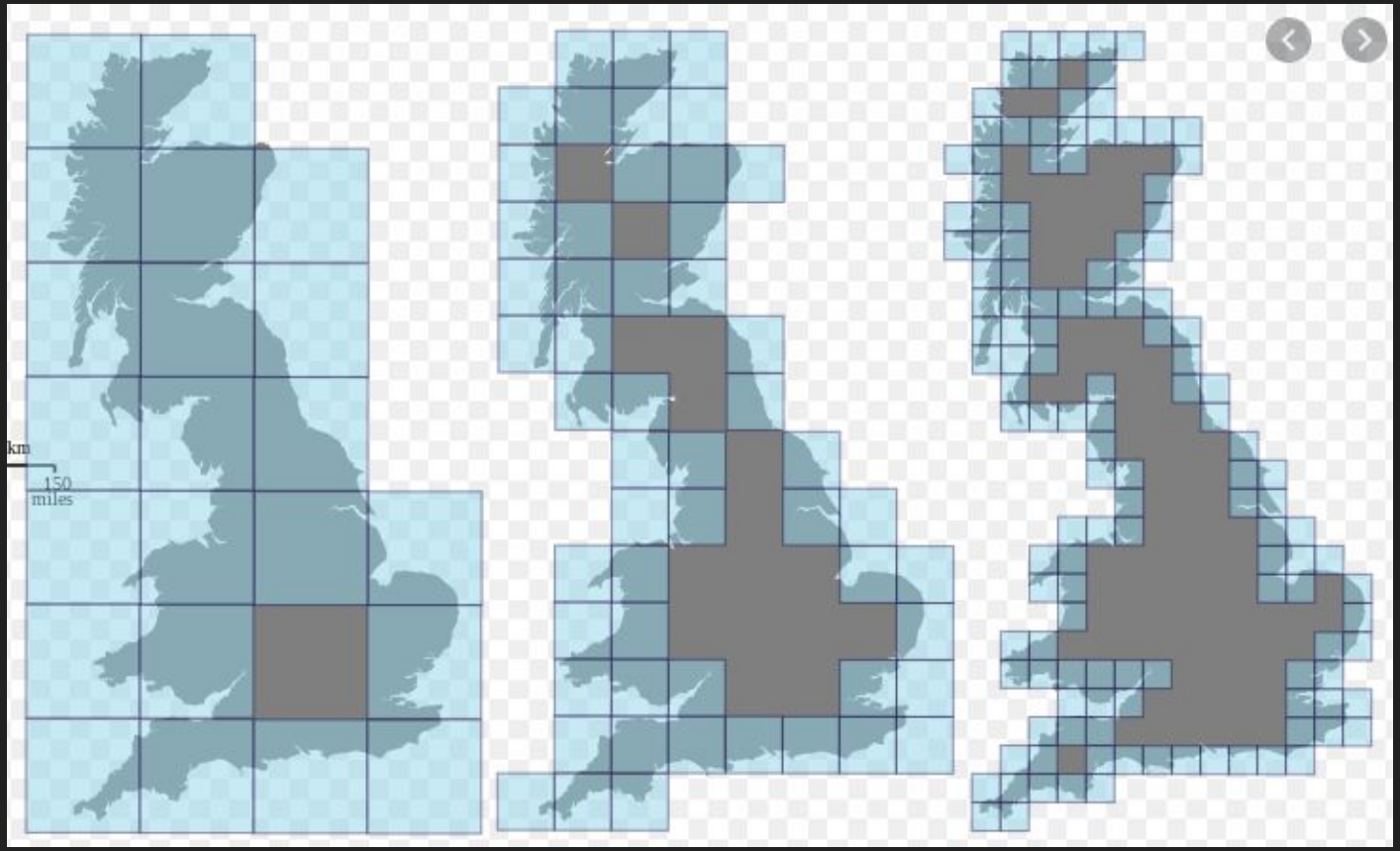


Von Koch Snowflake



Smooth

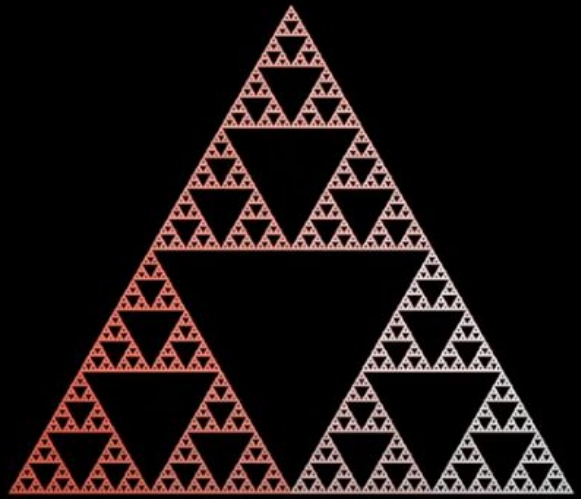
Fractal


km
150
miles

Definition of a fractal

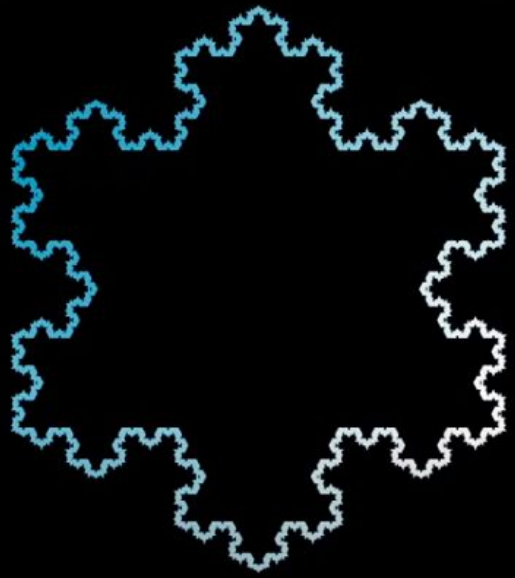
A fractal set is a set in metric space for which the Hausdorff-Besicovitch dimension is greater than the topological dimension.

$$D_F > D_T$$

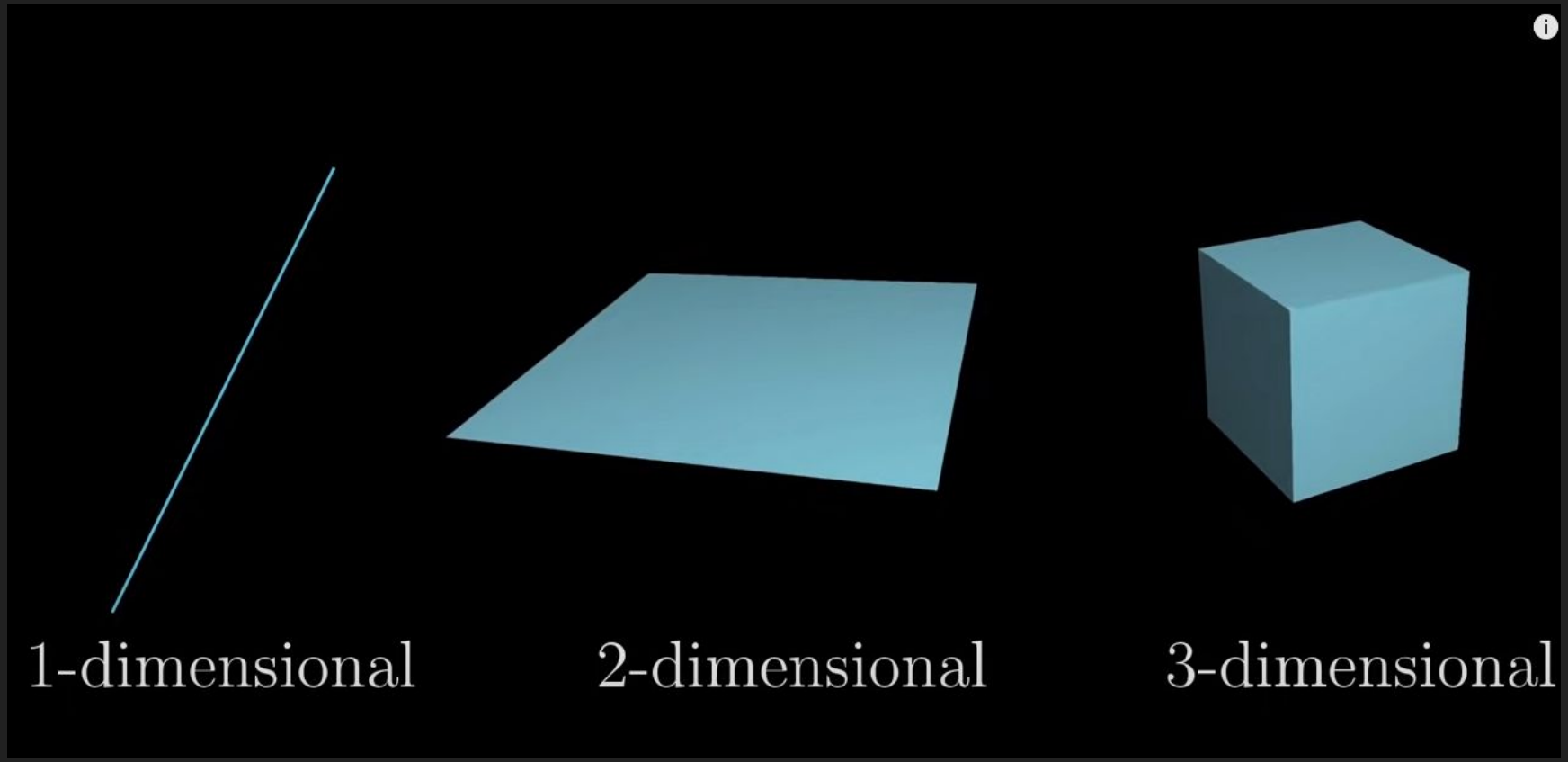


1.585-dimensional

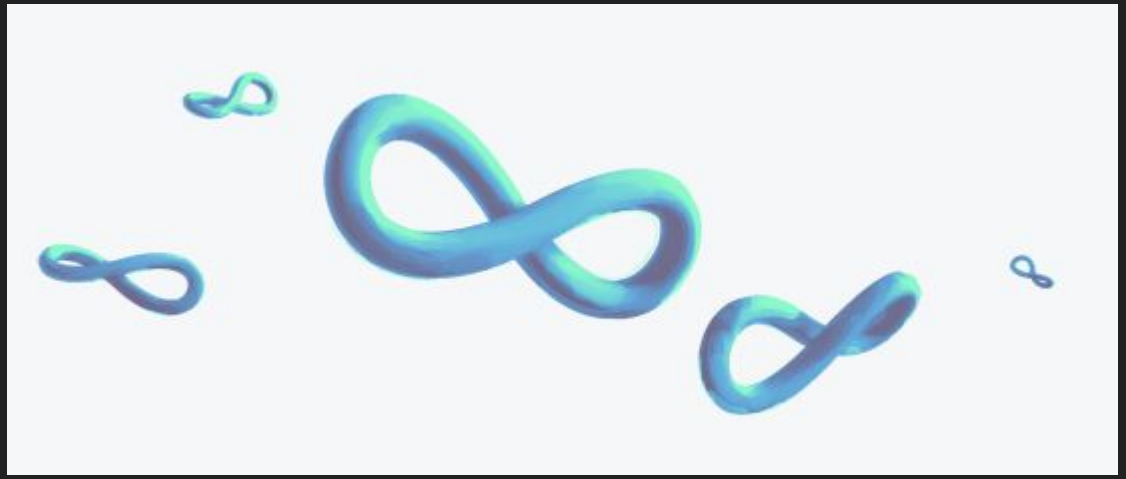
1.262-dimensional



1.21-dimensional


1-dimensional
2-dimensional
3-dimensional

The concept of ‘dimension’ is elusive and very complex, and is far from exhausted... However, the Hausdorff-Besicovitch dimension and the capacity dimension, when computed for random self-similar figures, have so far yielded the same value as the similarity dimension.



Shenker

1. Fractals are geometrically abnormal because they should be better seen as processes. This makes them ontologically and epistemologically more problematic than irrational numbers
2. Mandelbrot’s hypothesis that there is a fractal face to nature is empirically mistaken because it is based on so-called fractal images, not genuine fractals
3. Wherever fractal geometry yields useful approximations of natural forms, the system approximated is dominated by different scientific principles
4. The hypothesis of fractals is inconsistent with the atomistic paradigm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With regard to natural ferns, one can easily tell the scale that one is observing, with possible errors ranging between the few scales in which the fern is self-affine. But with regard to the fractal fern this is not possible, as all scales appear the same. One cannot tell whether one is observing the scale of one centimeter or of one Angstrom. True, such is the effect of self-affinity. But, it is not a natural effect. Therefore, the fact that the so-called fractal images resemble ferns is a result of their *finite* complexity, of the fact that they are self-affine in only a finite number of scales. In other words, they resemble natural forms due to their *not* being fractals.


Idealization
as smooth
Nature
Idealization
as perfectly
self-similar

I choose to define fractals, or see the necessary and sufficient condition for being a fractal, as objects having infinitely many details within a finite volume of the embedding space. Consequently, fractals have details on arbitrarily small scales, revealed as they are magnified or approached. Let us call this trait infinite complexity.

Plate 242 and 243 ▫ BROWN HULLS/ISLANDS; SELF-AVOIDING BROWNIAN MOTION

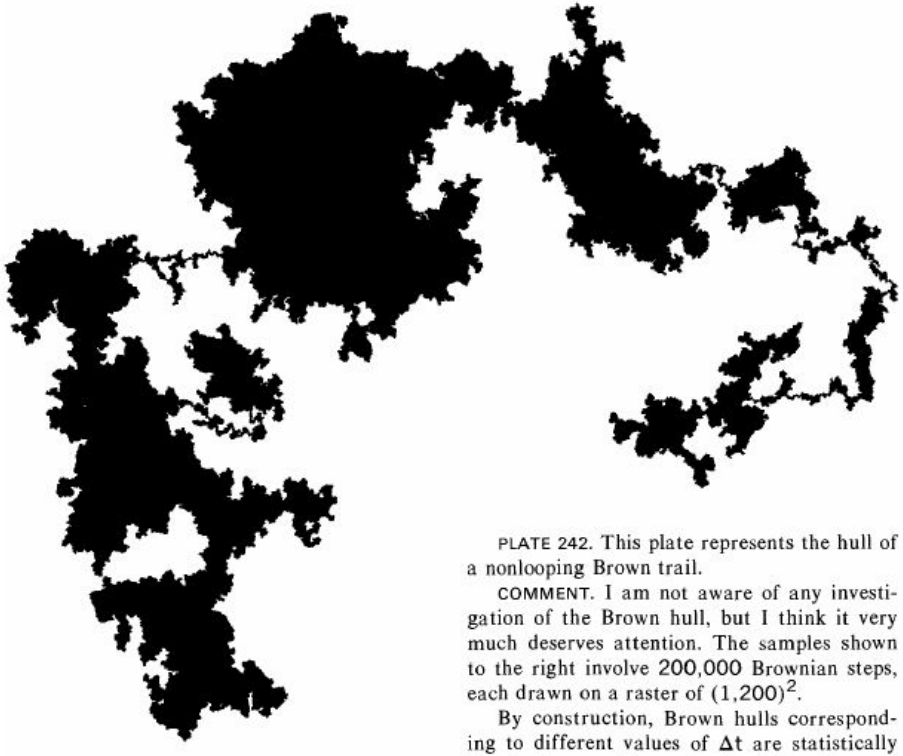


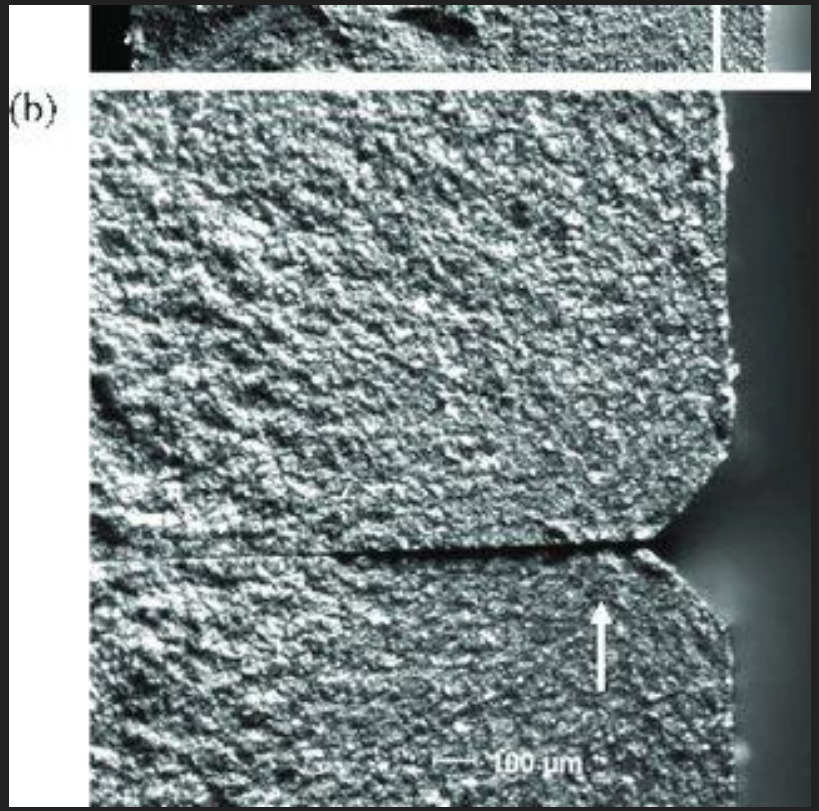
PLATE 242. This plate represents the hull of a nonlooping Brown trail.

COMMENT. I am not aware of any investigation of the Brown hull, but I think it very much deserves attention. The samples shown to the right involve 200,000 Brownian steps, each drawn on a raster of $(1{,}200)^2$.

By construction, Brown hulls corresponding to different values of Δt are statistically identical, except for scale. And there is every

Nottale

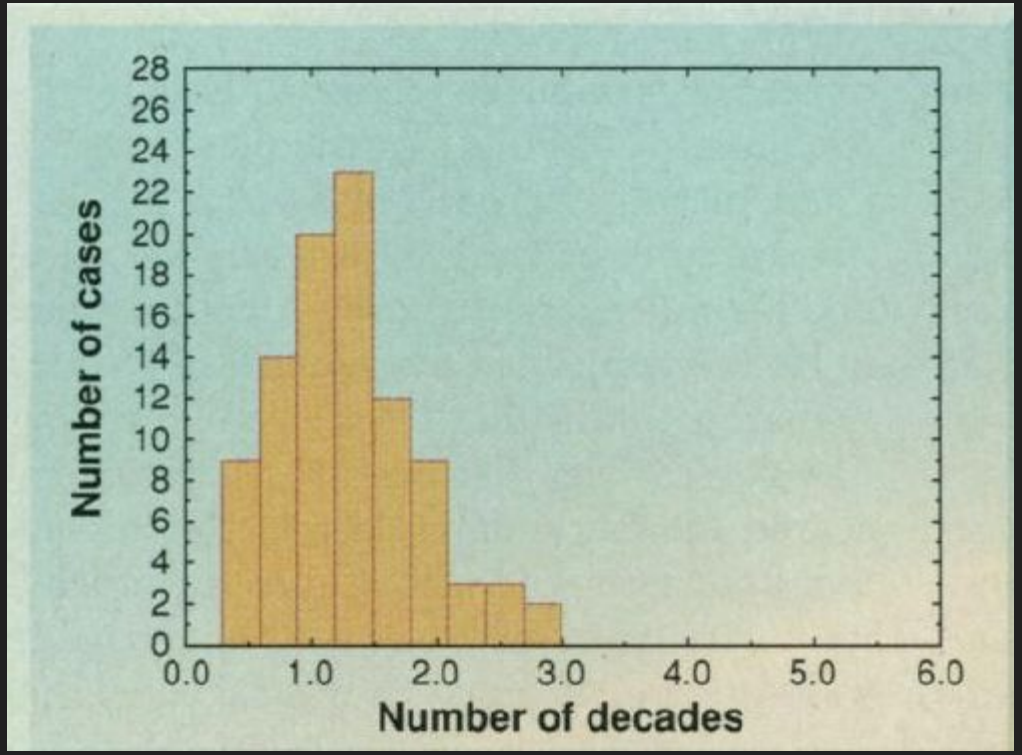
[As] stressed by Mandelbrot himself one should not identify fractality with self-similarity, which characterizes only some particular fractals and also apply to several non-fractal geometric shapes (in particular the most simple ones)


(b)
100 μm

Avnir et al.

Avnir et al. conducted a survey of ninety-six Physical Review articles in the years 1990-1996 explicitly using fractal analysis concepts and found that the average spread of resolutions probed was 1.3 orders of magnitude, with at most three orders of magnitude probed in one study.

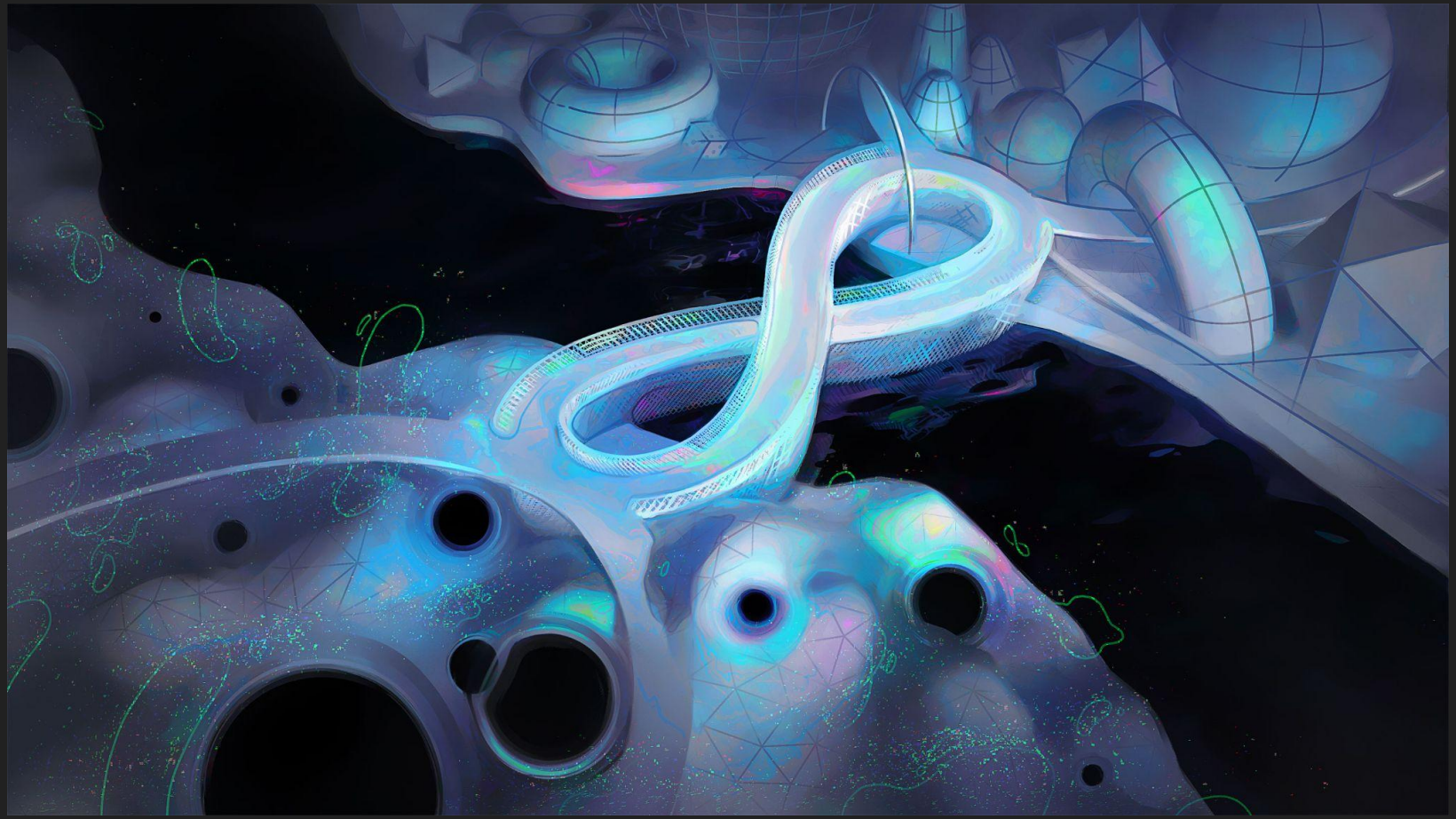
$$P = kr^{f(DF)}$$


Number of cases
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
0.0
1.0
2.0
3.0
4.0
5.0
6.0
Number of decades

“It may be too late to make any changes in a terminology that, at this stage, seems to be deeply rooted in practice. A drift from an original meaning of a concept is common in science, representing adaptability of the original ideal definition to realistic restrictions that emerge when put to practice.”

Recall the pragmatic principle

“Consider what effects, which might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object.”



References

Non-exhaustive due to them mostly being pictures. However, I owe important thanks and recognition to:

- 3blue1brown for the fractal visualizations
- Quanta Magazine for the QFT illustrations
- Mandelbrot for the Brownian motion statistically self-similar fractal
- Avnir et al, 1995
- Byrne’s book as reproduced online
- ...