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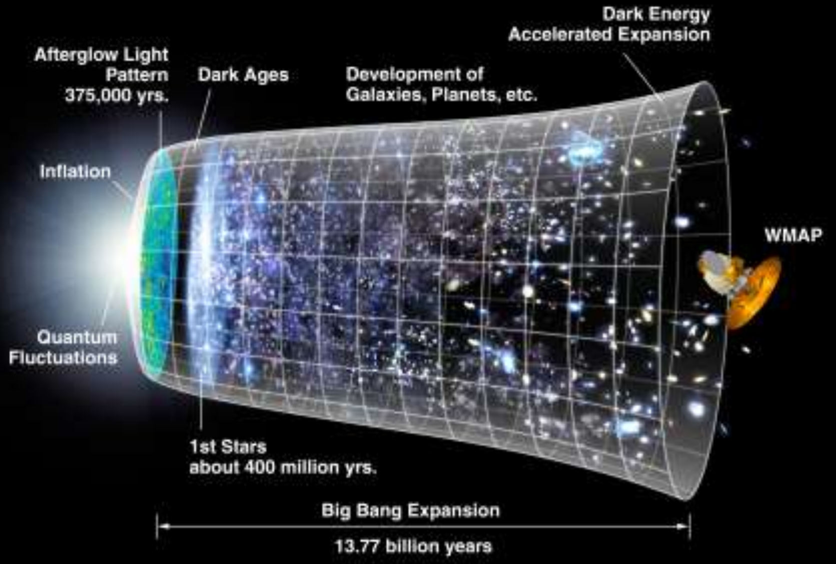
Memory and the Arrow of Time in Foundations of Quantum Mechanics

Sebastian Gil

Program: Bsc. in Physics and Astronomy

Supervisor: Dr. Ludovic van Waerbeke

FORECAST


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years

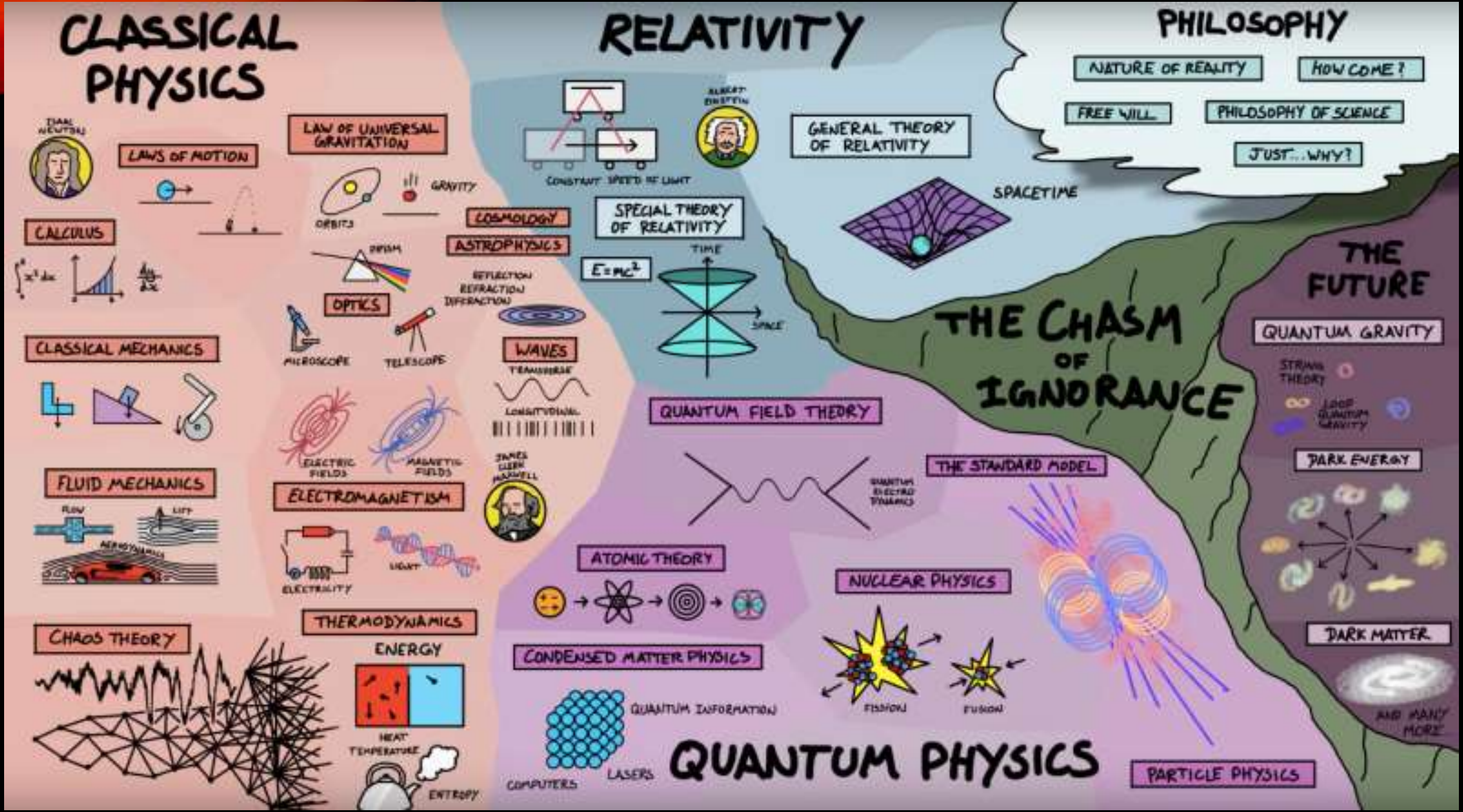
OUTLINE

- Motivation
- Statement of the Problem
- Methodology
- Results and Relevance
- Summary
- Future Work

MOTIVATION

OUTLINE

- Motivation
  - The Map of Physics
  - Conservation Laws are Fundamental
  - Shut up and Calculate!



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HOW COME?
FREE WILL
PHILOSOPHY OF SCIENCE
JUST...WHY?
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LAW OF UNIVERSAL GRAVITATION
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DARK MATTER
AND MANY MORE
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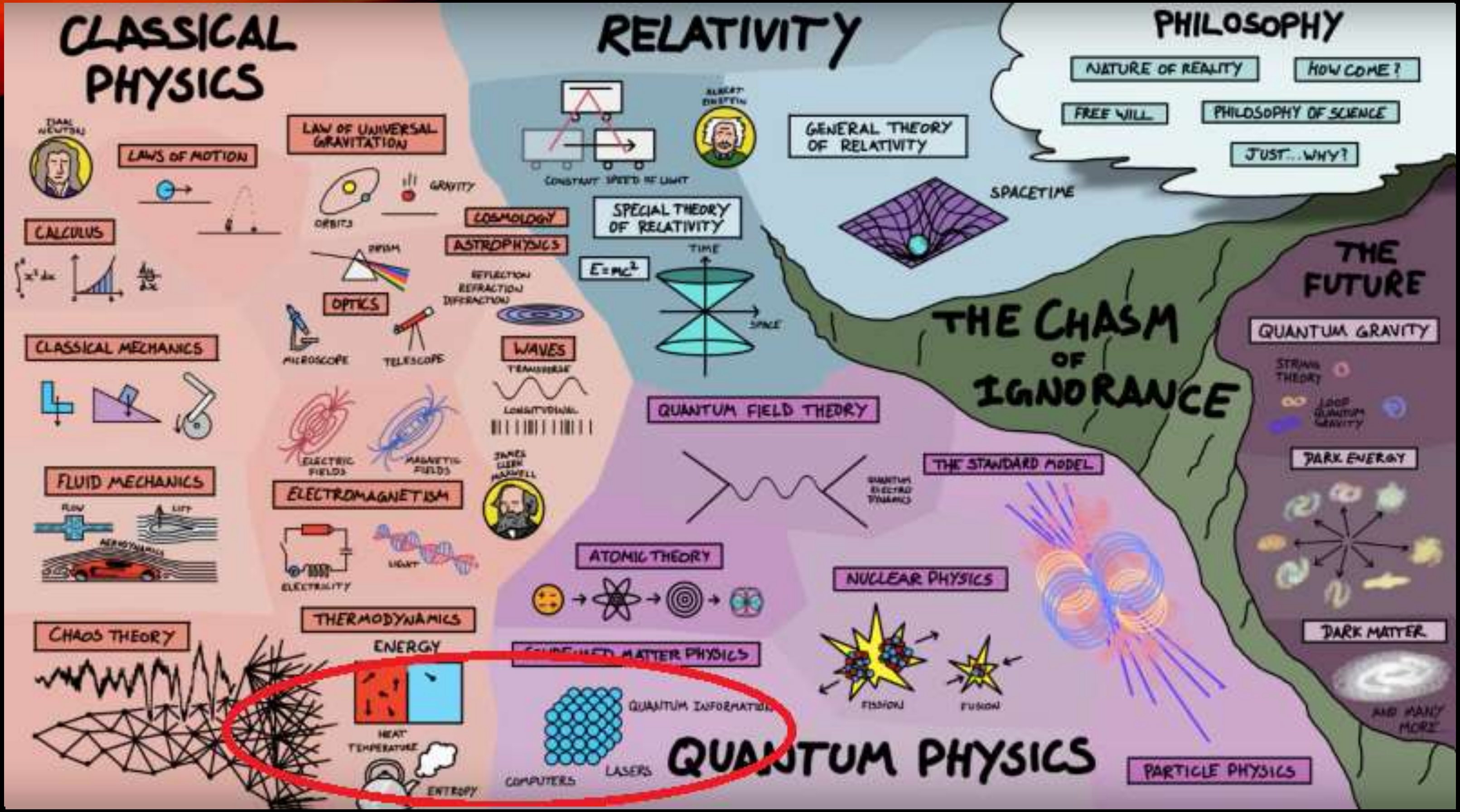

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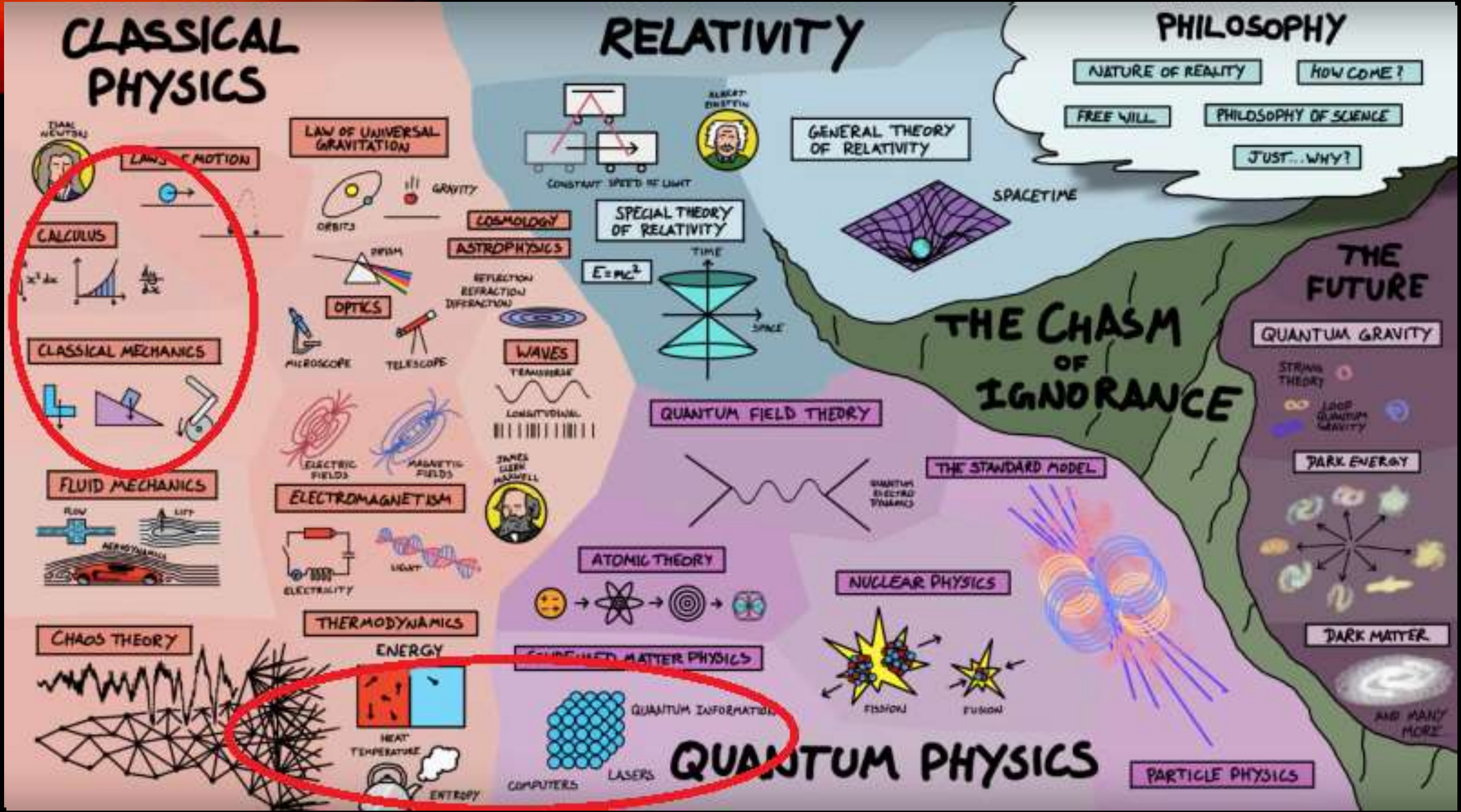
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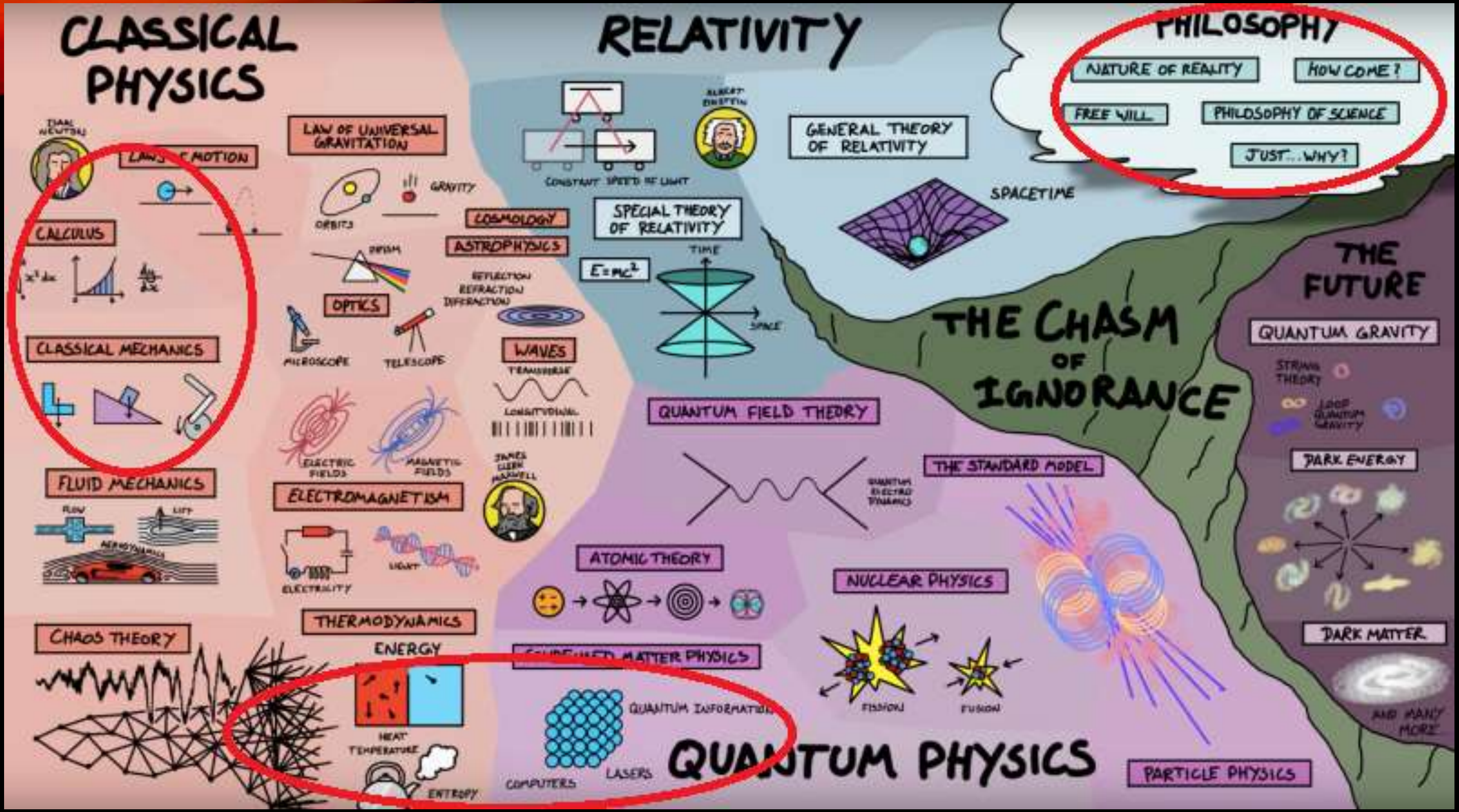

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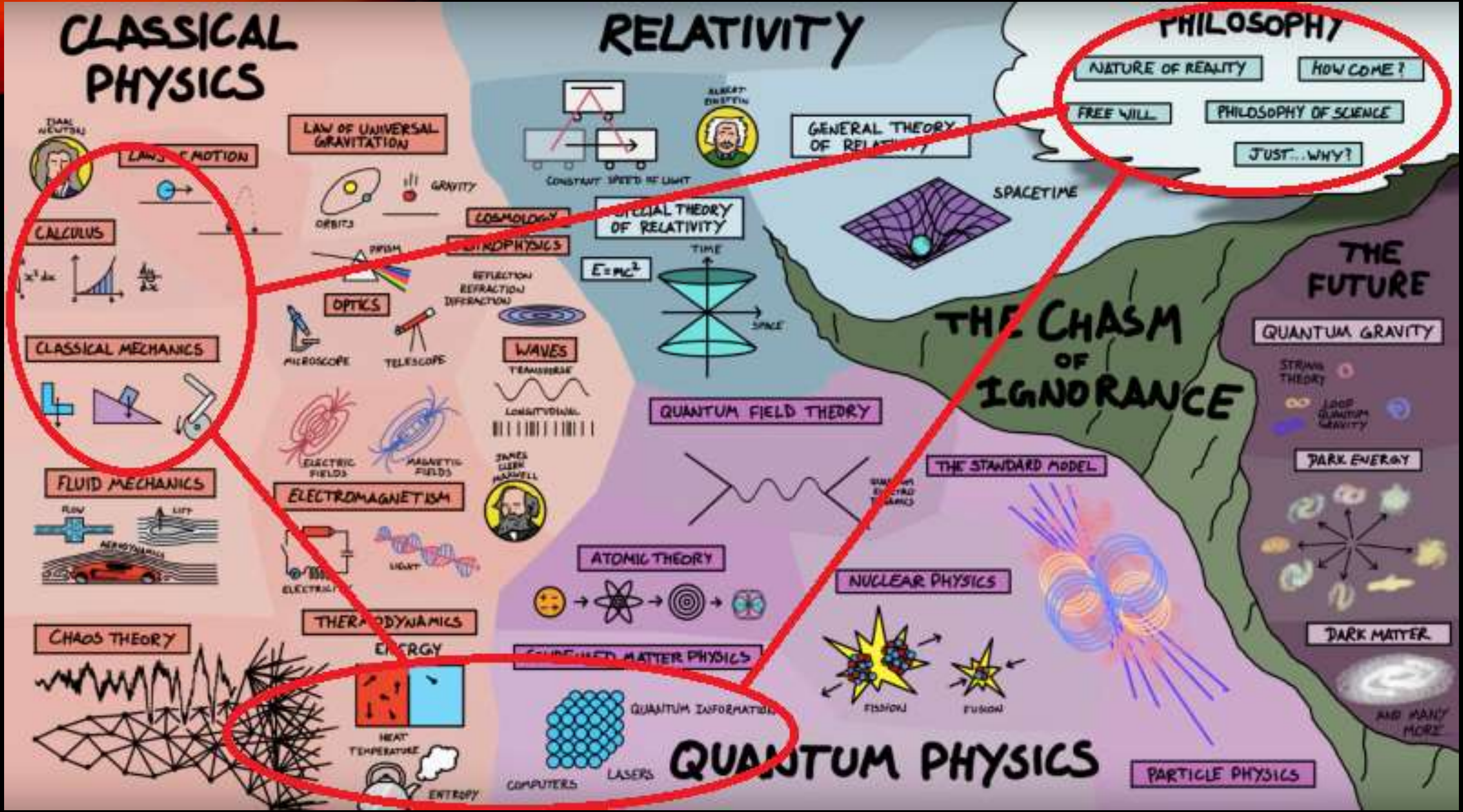

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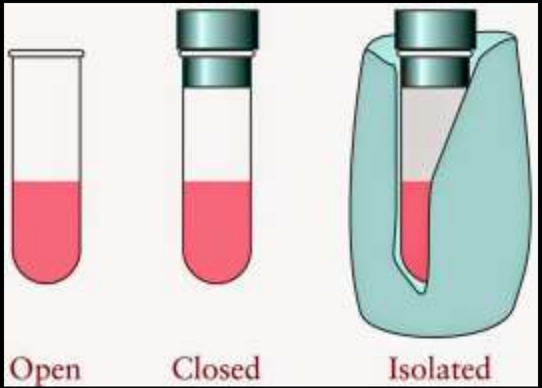
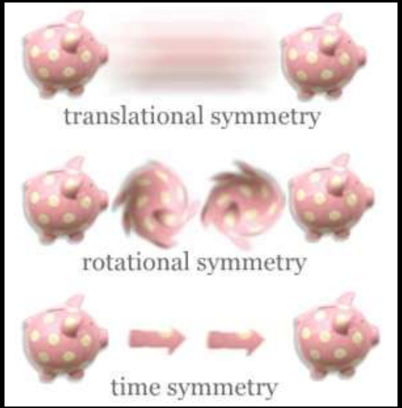
CONSERVATION LAWS ARE FUNDAMENTAL

- A conservation law is a statement that a particular measurable property of an **isolated** physical system will not change as the system evolves in time

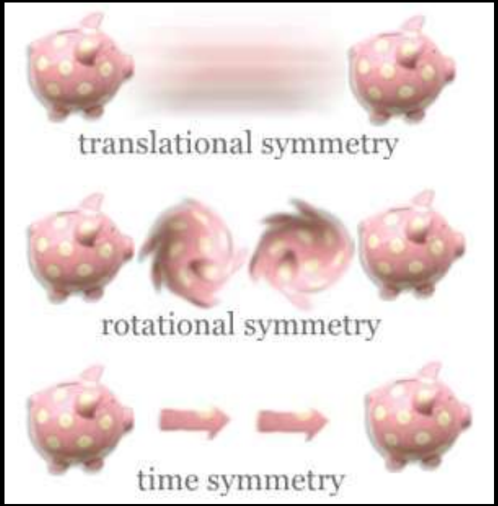




CONSERVATION LAWS ARE FUNDAMENTAL

- Classical mechanics relies on the conservation of three quantities:

- Linear momentum →
- Angular momentum →
- Energy →



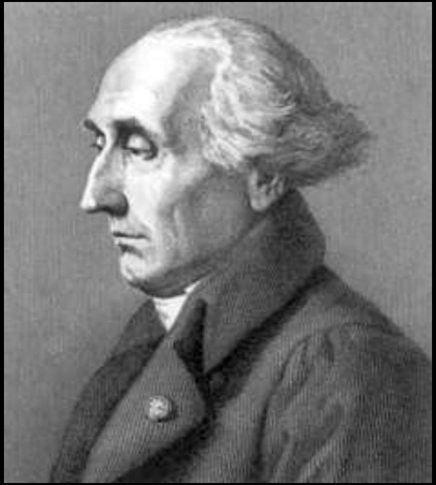
SHUT UP AND CALCULATE!

STATEMENT OF THE PROBLEM

OUTLINE

- Statement of the Problem
  - Different formulations of Classical Mechanics
  - Noether's Theorem
  - Dissipation
  - Where did the quantum friction go?
  - The Arrow of Time

THREE DIFFERENT FORMULATIONS OF CLASSICAL MECHANICS



- Newtonian Mechanics:
  - Standard 'high school physics'
  - Coincides with discovery of calculus
  - Calculations get tedious fast

- Sir Isaac Newton (1642-1726)

$$F_m = ma; \quad a = \frac{d^2x}{dt^2}$$

$$F_m = \frac{m}{g_c}\frac{d^2x}{dt^2}$$

$$\mathbf{v} = \mathbf{a}t + \mathbf{v}_0 \quad [1]$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \quad [2]$$

$$\mathbf{r} = \mathbf{r}_0 + \tfrac{1}{2}(\mathbf{v} + \mathbf{v}_0)\,t \quad [3]$$

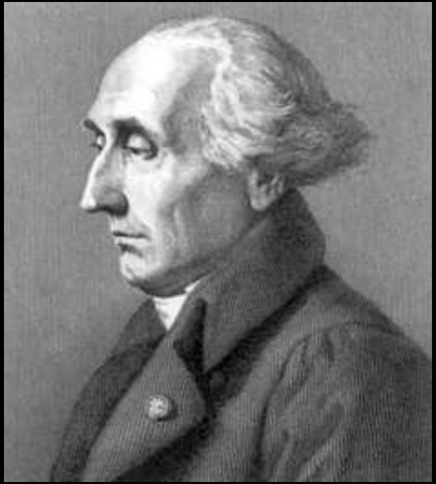
$$v^2 = v_0^2 + 2\mathbf{a}\cdot(\mathbf{r} - \mathbf{r}_0) \quad [4]$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t - \tfrac{1}{2}\mathbf{a}t^2 \quad [5]$$

$$\frac{\partial L}{\partial q} = \frac{\partial}{\partial q}(a\dot{q}^2 + bq^4) = 0 + \frac{\partial}{\partial q}bq^4 = 4bq^3$$
$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial}{\partial \dot{q}}(a\dot{q}^2 + bq^4) = \frac{\partial}{\partial \dot{q}}a\dot{q}^2 + 0 = 2a\dot{q}$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}} = \frac{\partial}{\partial t}2a\dot{q} = 2a\ddot{q}$$
$$\frac{\partial L}{\partial q} = \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{q}} \rightarrow 4bq^3 = 2a\ddot{q} \rightarrow \ddot{q} = \frac{2b}{a}q^3$$

- Lagrangian Mechanics:
  - Much more elegant
  - Works for all coordinates!
  - Exploits symmetries



- Joseph-Louis Lagrange (1736-1813)

- William Rowan Hamilton (1805-1865)

$$\begin{aligned}
\frac{\partial H}{\partial t} &= \sum_{i=1}^{n} \ddot{\vec{q}}_i \cdot \frac{\partial L}{\partial \dot{\vec{q}}_i} + \sum_{i=1}^{n} \dot{\vec{q}}_i \cdot \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{q}}_i}\right) - \frac{\partial L}{\partial t} \\
&= \sum_{i=1}^{n} \ddot{\vec{q}}_i \cdot \frac{\partial L}{\partial \dot{\vec{q}}_i} + \sum_{i=1}^{n} \dot{\vec{q}}_i \cdot \frac{\partial L}{\partial \vec{q}_i} - \frac{\partial L}{\partial t} \\
&= \frac{\partial L}{\partial t} - \frac{\partial L}{\partial t} \\
&= 0 - 0 \\
&= 0
\end{aligned}$$

- Hamiltonian Mechanics:
  - The “Hamiltonian” is the total energy
  - Is the **only** form of mechanics that we know how to **quantize**

$$
\begin{aligned}
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$$

- William Rowan Hamilton (1805-1865)

- Hamiltonian Mechanics:
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Conservation of energy!

$$
\begin{aligned}
\frac{\partial H}{\partial t} &= \sum_{i=1}^{n} \ddot{\vec{q}}_i \cdot \frac{\partial L}{\partial \dot{\vec{q}}_i} + \sum_{i=1}^{n} \dot{\vec{q}}_i \cdot \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\vec{q}}_i}\right) - \frac{\partial L}{\partial t} \\
&= \sum_{i=1}^{n} \ddot{\vec{q}}_i \cdot \frac{\partial L}{\partial \dot{\vec{q}}_i} + \sum_{i=1}^{n} \dot{\vec{q}}_i \cdot \frac{\partial L}{\partial \dot{\vec{q}}_i} - \frac{\partial L}{\partial t} \\
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$$

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- Hamiltonian Mechanics:
  - The "Hamiltonian" is the total energy
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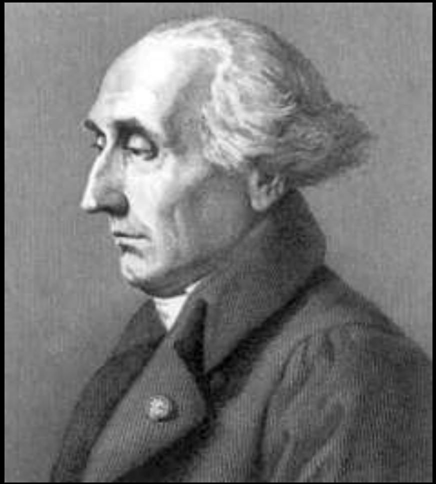
NOETHER'S THEOREM

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\mathbf{q}}}\frac{\partial \phi}{\partial \mathbf{q}}\dot{\mathbf{q}}T\right) &= \left(\frac{d}{dt}\frac{\partial L}{\partial \dot{\mathbf{q}}}\right)\frac{\partial \phi}{\partial \mathbf{q}}\dot{\mathbf{q}}T + \frac{\partial L}{\partial \dot{\mathbf{q}}}\left(\frac{d}{dt}\frac{\partial \phi}{\partial \mathbf{q}}\right)\dot{\mathbf{q}}T + \frac{\partial L}{\partial \dot{\mathbf{q}}}\frac{\partial \phi}{\partial \mathbf{q}}\ddot{\mathbf{q}}T \\ &= \frac{\partial L}{\partial \mathbf{q}}\frac{\partial \phi}{\partial \mathbf{q}}\dot{\mathbf{q}}T + \frac{\partial L}{\partial \dot{\mathbf{q}}}\left(\frac{\partial^2 \phi}{(\partial \mathbf{q})^2}\dot{\mathbf{q}}\right)\dot{\mathbf{q}}T + \frac{\partial L}{\partial \dot{\mathbf{q}}}\frac{\partial \phi}{\partial \mathbf{q}}\ddot{\mathbf{q}}T.\end{aligned}$$

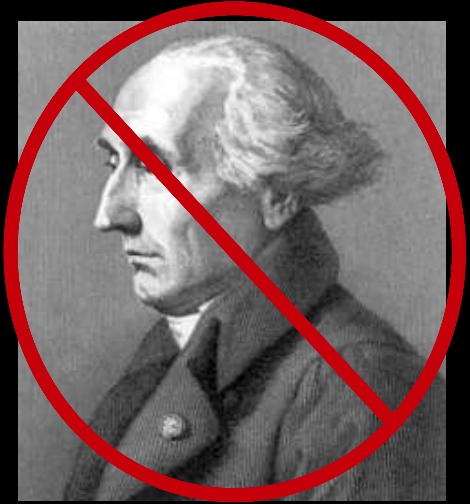
-
- Symmetries $\longleftrightarrow$ Conservation Laws!

DISSIPATION

- Dissipative systems do not obey conservation laws
  - Newtonian mechanics can deal with friction
  - Lagrangian and Hamiltonian mechanics cannot



- Dissipative systems do not obey conservation laws
  - Newtonian mechanics can deal with friction
  - Lagrangian and Hamiltonian mechanics **cannot**



- But quantum mechanics is derived explicitly from Hamiltonian mechanics...

- So are quantum systems truly conservative and therefore time reversible?
-
-
-

- OR

- So are quantum systems truly conservative and therefore time reversible?
-
- OR
-
- We don't know how to incorporate dissipation into quantum systems?

WHERE DID THE QUANTUM FRICTION GO?

-
- Statement of the problem: the foundations of quantum mechanics are explicitly time reversible, but this is only true for isolated conservative systems!
-

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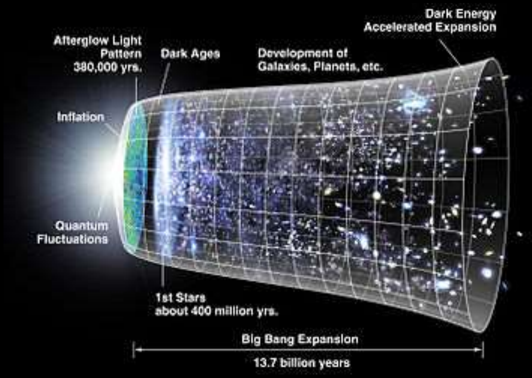
- We live in a world dominated by friction and irreversible dissipative processes

WHERE DID THE QUANTUM FRICTION GO?

- Statement of the problem: the foundations of quantum mechanics are explicitly time reversible, but this is only true for isolated conservative systems!
- We live in a world dominated by friction and irreversible dissipative processes
- In thermodynamics, this is known as...

THE ARROW OF TIME



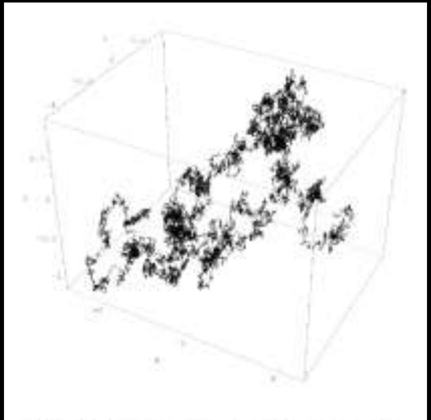
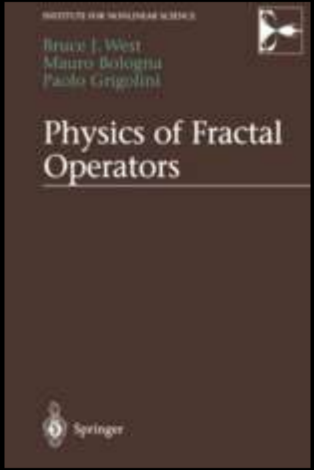


FIG. 4. The Brownian motion of a particle can be modeled in terms of a random walk. The curve depicted is a three-dimensional representation of the particle's path.[15]

METHODOLOGY

OUTLINE

- Methodology
  - Expanding the Mathematical Toolkit
  - Identifying experimental scenarios that are intrinsically dissipative
  - Instead on relying on computer simulations, focus on analytic approaches
    - Striving for conceptual clarity!



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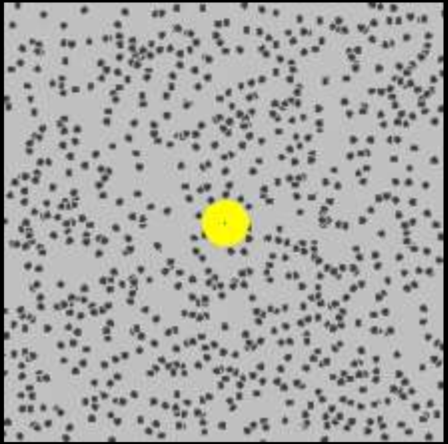
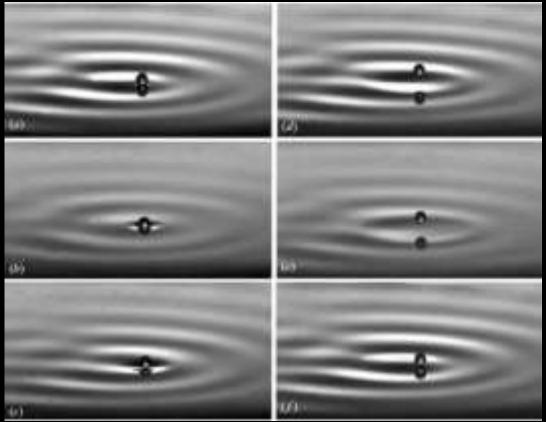
MATLAB

The Language of Technical Computing





RESULTS

OUTLINE

- Results (so far)
  - Nonconservative Lagrangian Mechanics
  - Fractional Calculus and Memory
  - Perturbation Expansion
    - Small deviations from classical trajectories
  - Is space-time… a fractal?

NONCONSERVATIVE LAGRANGIAN MECHANICS

Nonconservative Lagrangian Mechanics: Derivation of a Variational Principle for Linear Friction

A Fractional Calculus Approach

by

Sebastián Gil

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

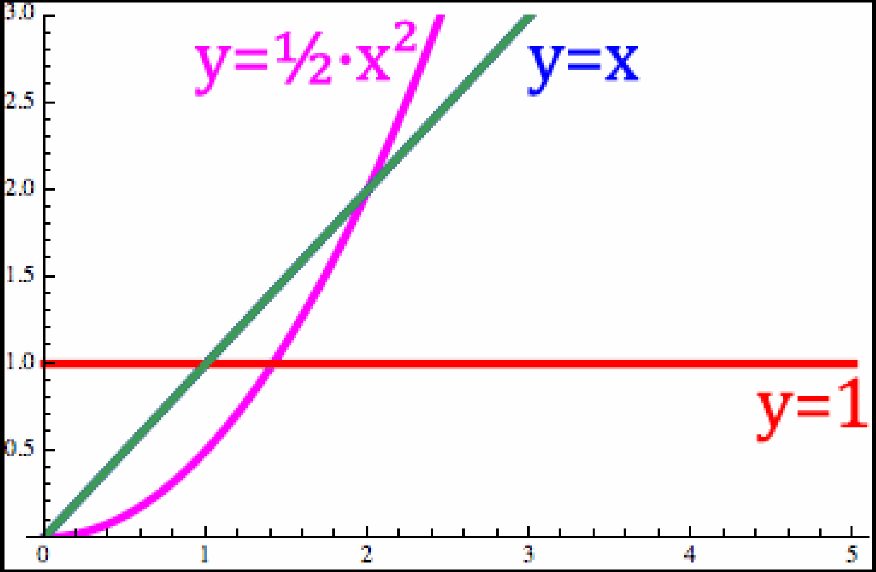
BACHELLOR OF SCIENCE

in

Faculty of Science

(Combined Honours in Physics and Astronomy)

FRACTIONAL CALCULUS



- A fractional derivative is:
  - an extension of regular derivatives from whole numb values to any real number
  -

2.2.4 The Fractional Derivative

The Riemann-Liouville definition of the fractional derivative follows naturally from the fractional integral. Choosing an arbitrary order of integration $\alpha > 0$, we first need to find the smallest integer m exceeding α. That is to say, we need to choose m such that

$$m - 1 < \alpha < m. \tag{2.23}$$

Then, the fractional derivative is defined as

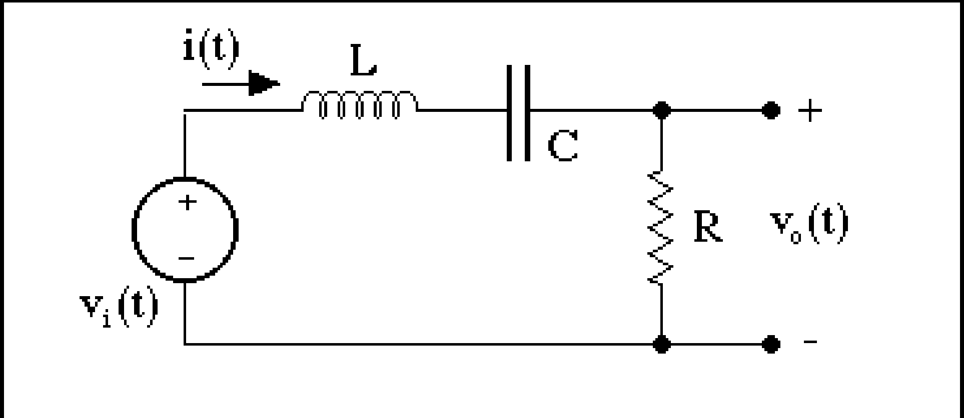
$$ {}_aD_t^\alpha[f(t)] = \frac{d^m}{dt^m}\left[\frac{1}{\Gamma(m-a)}\int_a^t f(\tau)(t-\tau)^{m-\alpha-1}\,d\tau\right] \tag{2.24}$$

Defined this way, we ensure that the Gamma function does not become undefined. To clarify, the fractional derivative of an appropriately chosen function $f(t)$ is computed in the following way:

1. Choose desired order of differentiation α
2. Find $m : m - 1 < \alpha < m$
3. Integrate $f(t)$ to $m - \alpha$ order
4. Differentiate the result m times

Defined this way, it can easily be verified that the Riemann-Liouville fractional derivative reduces back to the standard derivative when α is chosen to be an integer.

SYSTEMS WITH MEMORY



3.5 Application to Linear Friction

In compliance with Riewe's argument, let $\alpha = 1/2$ and consider the Lagrangian

$$L = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 + \frac{C}{2}[{}_bD_t^{1/2}q]^2 - V(q). \tag{3.32}$$

Here the first term denotes the kinetic energy of the particle while the second term corresponds to a 'linear friction energy.' The potential in the third term remains unspecified. Inserting this Lagrangian into (3.31) gives

$$\begin{aligned} 0 &= -\frac{\partial V}{\partial q} + {}_bD_t^{1/2}[C_bD_t^{1/2}q] + {}_bD_t^1[m\dot{q}] && (3.33) \\ &= -\frac{\partial V}{\partial q} + C\dot{q} + m\ddot{q} && (3.34) \end{aligned}$$

where we obtained the second term from the composition rule of fractional derivatives and the third term from identifying the differentiation operator to be that of a first derivative with respect to time. If we now let $V(x) = \frac{1}{2}m\omega^2x^2$, the resulting equation of motion is

$$m\ddot{q} + C\dot{q} - m\omega^2q = 0 \tag{3.35}$$

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$$\begin{aligned} 0 &= -\frac{\partial V}{\partial q} + {}_bD_t^{1/2}[C\,{}_bD_t^{1/2}q] + {}_bD_t^1[m\dot{q}] && (3.33) \\ &= -\frac{\partial V}{\partial q} + C\dot{q} + m\ddot{q} && (3.34) \end{aligned}$$

where we obtained the second term from the composition rule of fractional derivatives and the third term from identifying the differentiation operator to be that of a first derivative with respect to time. If we now let $V(x) = \frac{1}{2}m\omega^2x^2$, the resulting equation of motion is

$$m\ddot{q} + C\dot{q} - m\omega^2q = 0 \tag{3.35}$$

3.5 Application to Linear Friction

In compliance with Riewe's argument, let $\alpha = 1/2$ and consider the Lagrangian

$$L = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 + \frac{C}{2}[{}_bD_t^{1/2}q]^2 - V(q). \tag{3.32}$$

Here the first term denotes the kinetic energy of the particle while the second term corresponds to a 'linear friction energy.' The potential in the third term remains unspecified. Inserting this Lagrangian into (3.31) gives

$$\begin{aligned} 0 &= -\frac{\partial V}{\partial q} + {}_bD_t^{1/2}[C\,{}_bD_t^{1/2}q] + {}_bD_t^1[m\dot{q}] && (3.33)\\ &= -\frac{\partial V}{\partial q} + C\dot{q} + m\ddot{q} && (3.34)\end{aligned}$$

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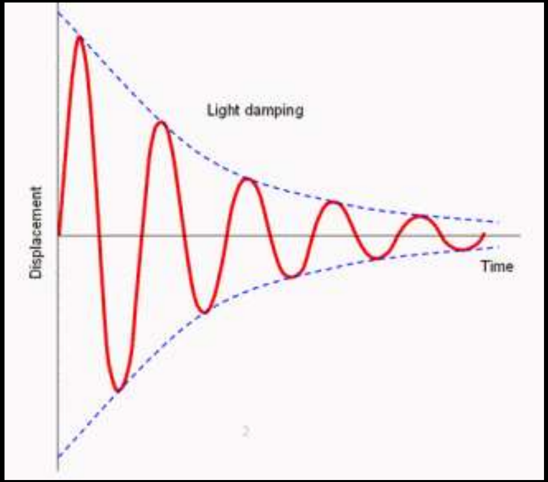
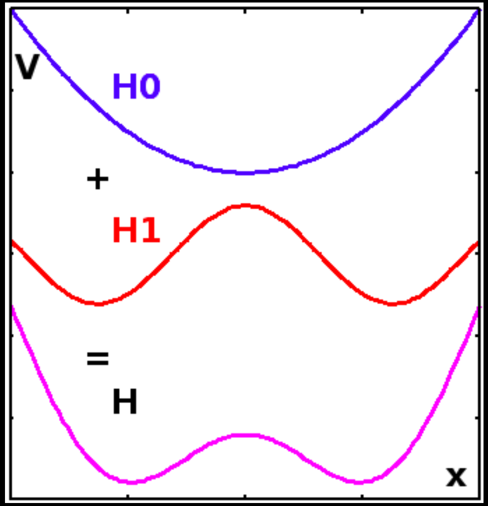
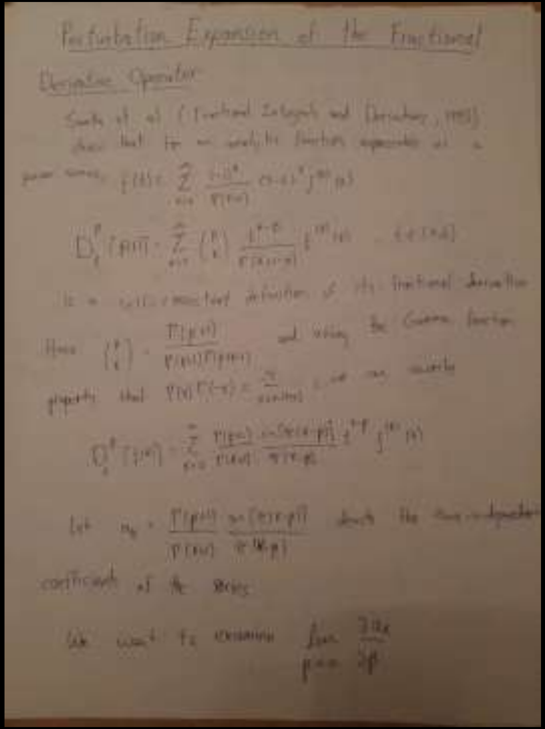
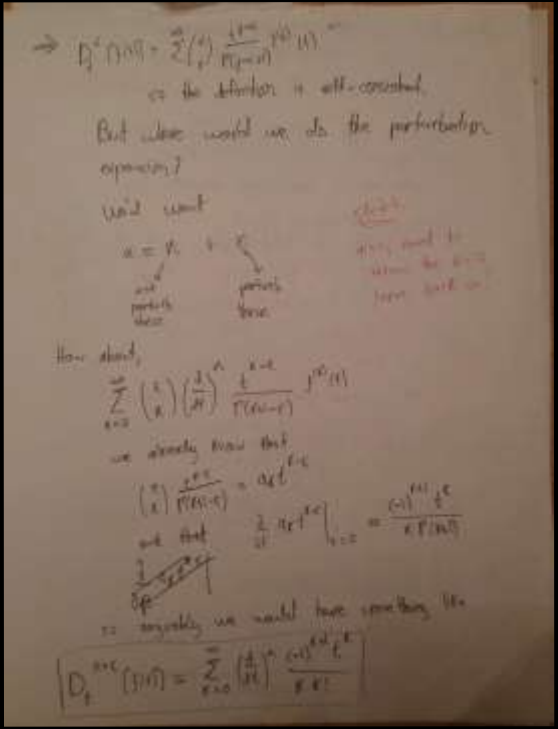
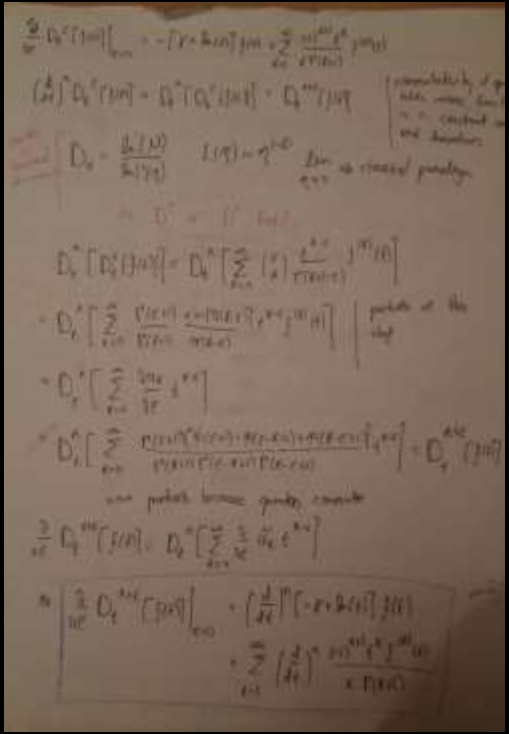


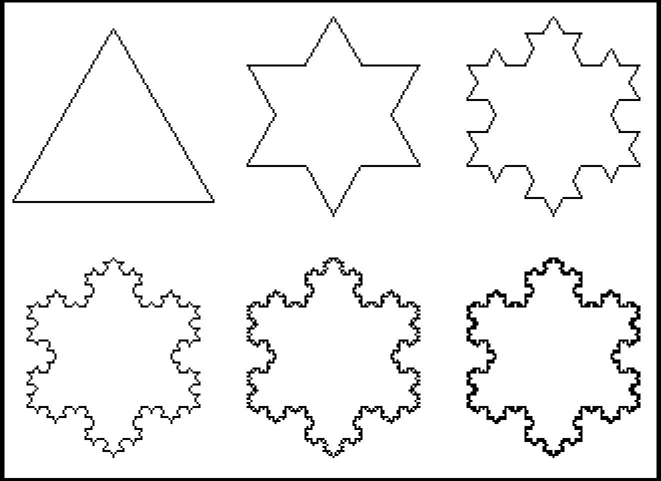
PERTURBATION EXPANSION

- Widely used approach to find how much quantities deviate from a simple case
- In this context, deviation from ‘straight line’ trajectories

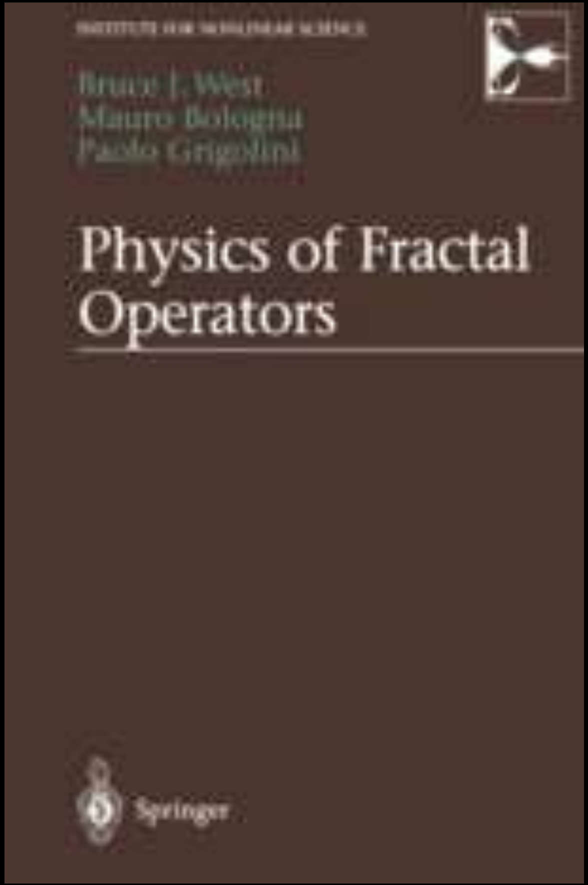



Perturbation Expansion of the Fractional
Derivative Operator
coefficients of the series

But where would we do the perturbation
expansion?
How about,
we already know that

we perturb because

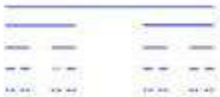
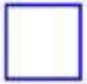
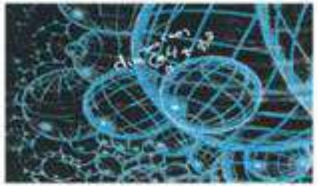
FRACTAL GEOMETRY



-
- A fractal is any geometric structure that exhibits a **self-similar** relation



Bruce J. West
Mauro Bologna
Paolo Grigolini
Physics of Fractal Operators
Springer

Type of fractal	Geometrical shape	Menger–Urysohn dimension	Hausdorff dimension	Corresponding random Hausdorff dimension	Embedding dimension	Corresponding Euclidean shape
Cantor Set		0	*ln 2/ln3 = 0.630929753*	$\phi = 0.61803398$	1	 *Line*
Sierpinski gasket		2	*ln 3/ln2 = 1.584962501*	$\frac{1}{\phi} = 1.618033989$	2	 *Square*
Menger sponge		3	D_{MS} = *ln20/ln3 = 2.7268*	$2 + \phi =$ 2.61803398	3	 *Cube*
The 4 dimension random cantor set analogue of Menger sponge	 *An artist impression of* ε[illegible] *space-time*	4	$d_c^{(4)} =$ 4.236068	$4 + \phi^3 =$ 4.23606797	5	 *Hyper cube*

RELEVANCE

- Relevance
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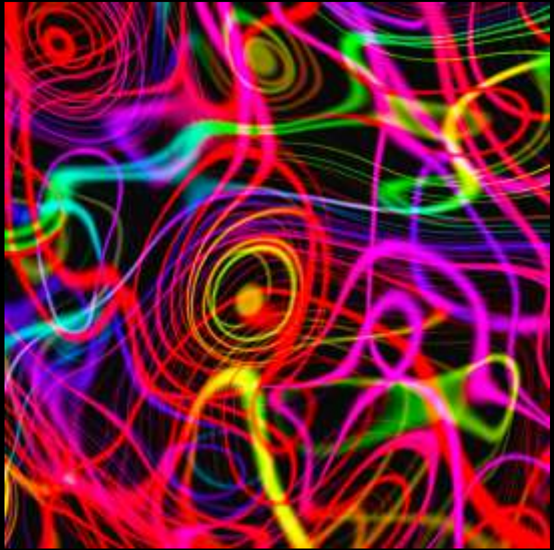
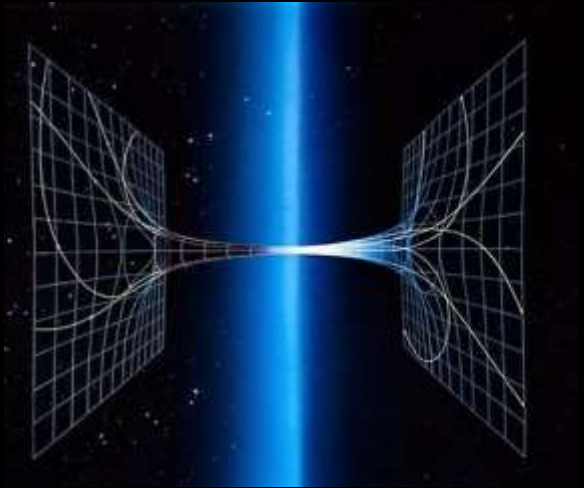
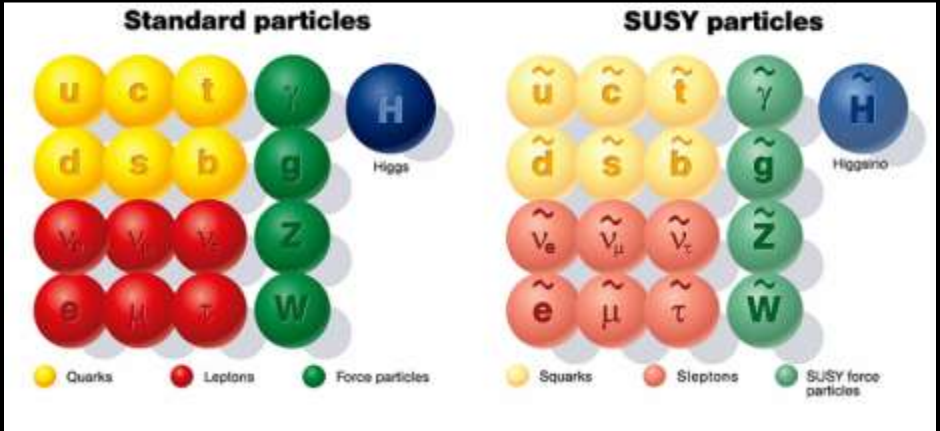
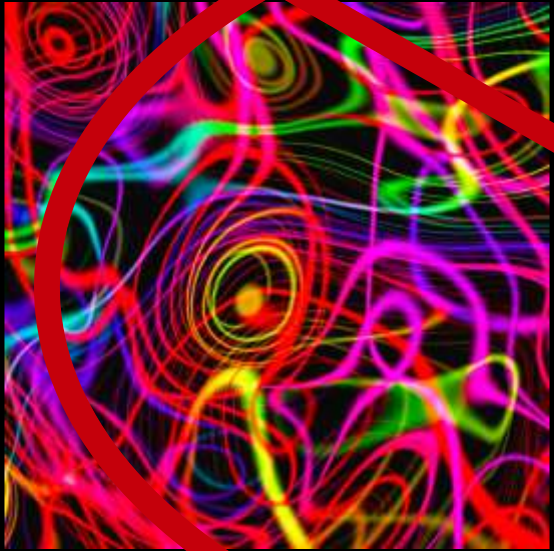
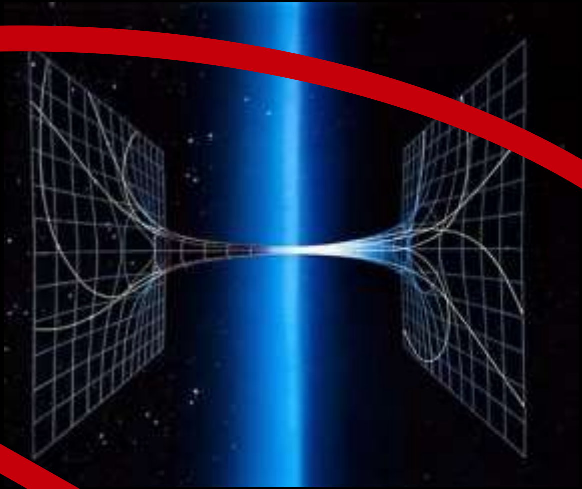
  - Motivate research programs on fundamental physics beyond the current paradigm of supersymmetry and string theory
  - Apply theoretical framework to experimental settings
  - Shut up and calculate!

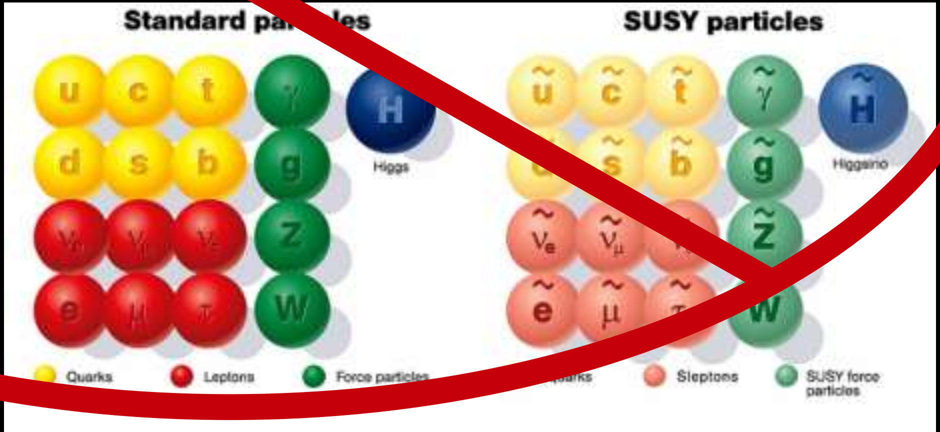





Standard particles
SUSY particles
Higgs
Higgsino
Quarks
Leptons
Force particles
Squarks
Sleptons
SUSY force particles






SUSY particles
Higgs
Higgsino
Quarks
Leptons
Force particles
Sleptons
SUSY force particles

SUMMARY

FUTURE WORK

OUTLINE

- Future Work
  - Test new paradigm against experimental data:
    - Brownian motion
    - Bouncing Droplets
    - Many Body Localization

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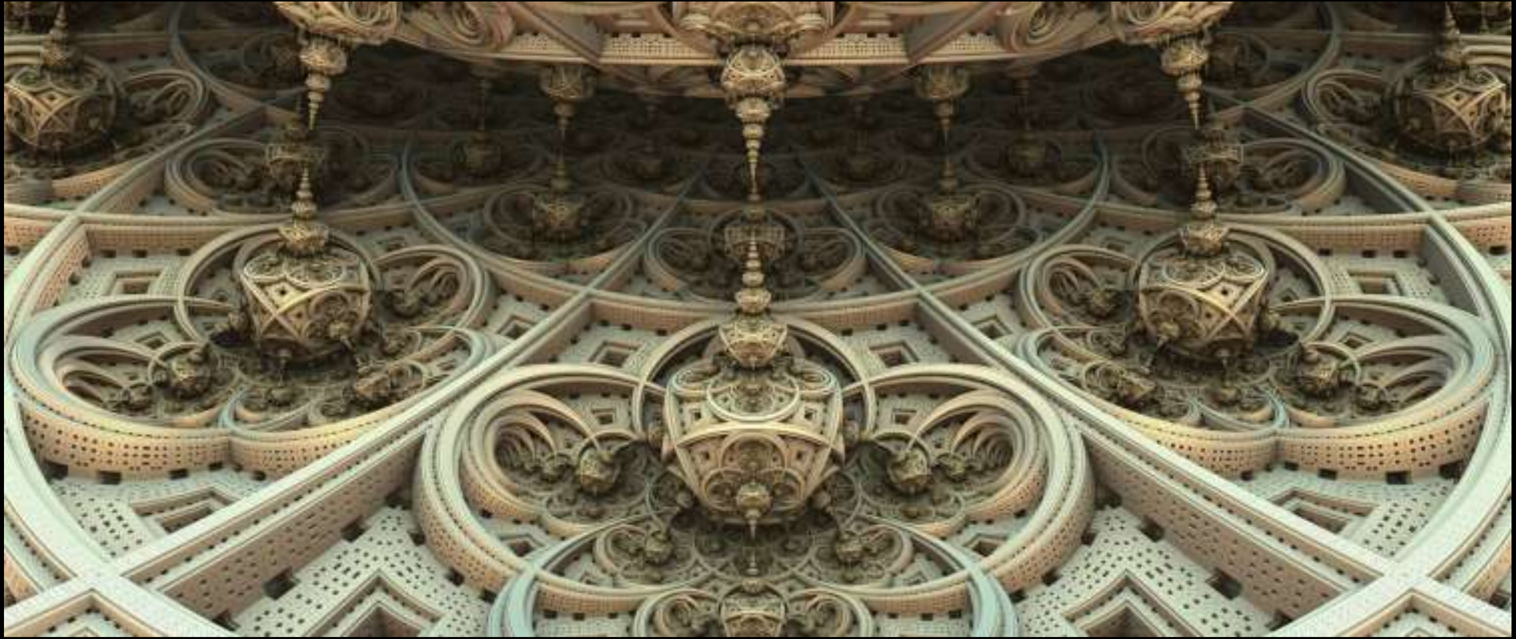
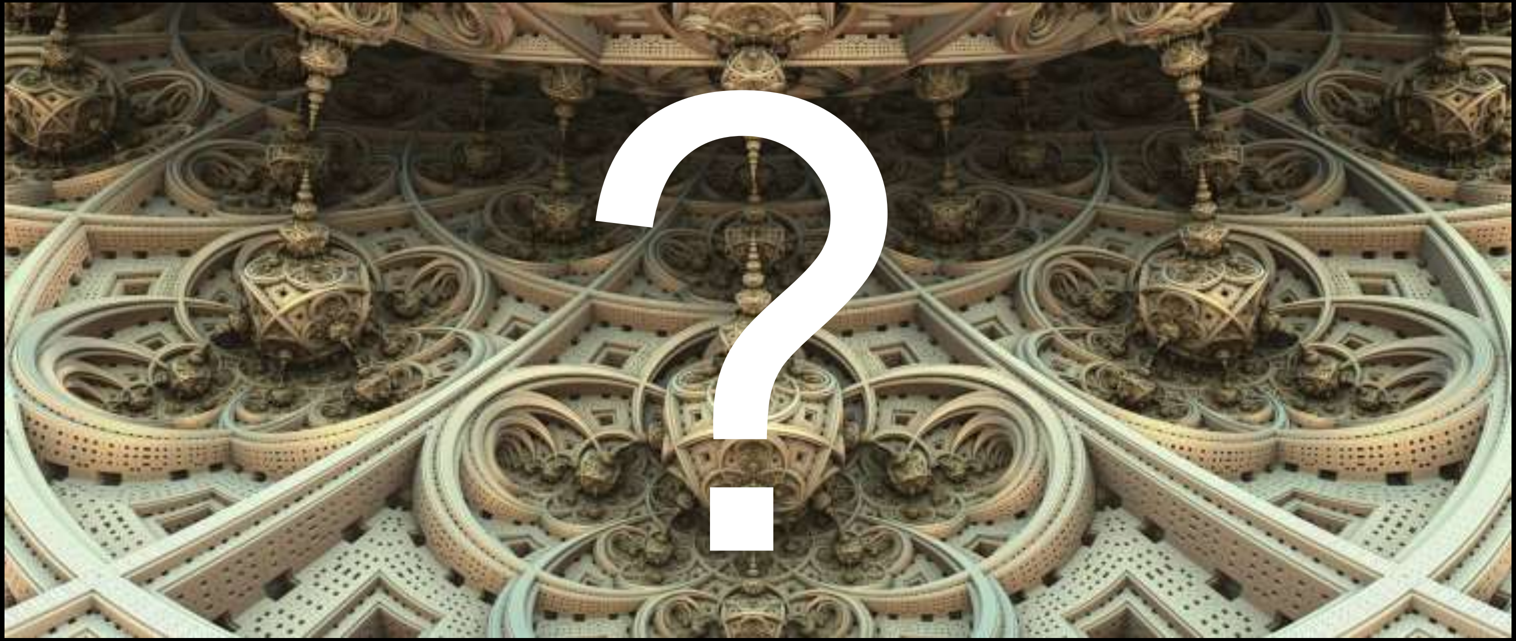
- Future Work
  - Test new paradigm against experimental data:
    - Brownian motion
    - Bouncing Droplets
    - Many Body Localization
  - Develop numerical and computational models
  - How will my models fare in comparison to modern canonical theory, namely Quantum Field Theory?





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